Implicatures in Continuation-Based Dynamic Semantics by Reasoning in the Context

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Except where otherwise indicated, this thesis is my own original work.

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Abstract

The translation of natural language expressions into logical representations provides a formal semantics for linguistic study and contributes to natural language processing applications. Amongst current formalisms for this task, de Groote [2006]'s continuation-based dynamic semantics – as systematized and extended by Lebedeva [2012] – distinguishes itself by returning to the principles of the first extensive formal semantics for natural language (Montague [1970a,b, 1973]): using only standard tools from mathematical logic in a fully compositional way, it captures phenomena such as cross-sentential pronominal anaphora, quantifier scope and presupposition projection. Discourse context is incorporated as a parameter, meaning its structure may be changed while preserving much of the operation of the framework. Exploiting this feature, I define a more elaborate context structure to allow for basic commonsense reasoning, and show how this can be used to capture implicature-related content with typical instances from three classes of meaning: Grice [1975]'s conversational implicatures; Grice [1975]'s conventional implicatures, with a focus on *but*; and the 'CI' of Potts [2005b], specifically supplementary content. I do this in the spirit of using only common tools from mathematical logic by adapting Poole's framework for default and abductive reasoning (Poole [1988, 1989, 1990]), a way of using classical logic for commonsense reasoning by viewing reasoning as theory formation. I situate this work within Potts [2015] call for shifting focus from "splitting and lumping" into meaning classes to "rich theories of properties" by suggesting formal definitions of properties of these meaning classes, in a formalism that is now capable of a range of presuppositions, conversational implicatures, conventional implicatures, and CIs.

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Chapter 1

Introduction

This is a thesis from logic, in natural language semantics. Symbolic methods contribute to linguistic enquiry by enabling formal descriptions and comparisons of natural language phenomena. They can also advance natural language processing for human-computer interfaces – despite the success of statistical methods for this task, limitations in data-driven approaches exist in areas where symbolic methods excel, such as negation and antinomy [de Paiva, 2011, p.86].

The problem addressed in this thesis is formalizing meaning associated with *implicatures*, which roughly refers to meaning outside of what is explicitly stated. By accounting for implicatures in a logical language, categorizations within this meaning class and comparisons to similar phenomena can be made precise. Not any formalism will do: to make it accessible and implementable, it should use only standard tools from logic and be fully compositional – the meaning of a larger unit is built from the meaning of its smaller units, in a systematic way.

To this end, this thesis builds on the semantics of de Groote [2006] as extended by Lebedeva [2012], changing the structure of the context of background knowledge and preceding discourse with respect to which sentences are interpreted. By modelling the context as a logic of commonsense reasoning, implicature-related meaning is located and its properties analysed. In doing so, this research contributes to Potts' call to move from "splitting and lumping" meaning into different categories towards developing "rich theories of properties... the way those properties interact, and the effects of those interactions on language and cognition." [Potts, 2015, p.36]

This chapter proceeds by elaborating the context of the problem and situating the contributions of this thesis. A complete background will be given in Chapter 2.

1.1 Context

Before the rise of natural language processing and its technological applications, natural language and computer science intersected in the work of mathematician Richard Montague, who made the following contention contrary to the prevailing view of the time:

> There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians. [Montague, 1970b, p.22]

Montague [1970a,b, 1973] assigns to each word in a fragment of English a formula in a version of Church [1940]'s simply typed λ-calculus; this logical form is referred to as the *interpretation* of the word. Montague [1970b] continues:

> ... indeed, I consider it possible to comprehend the syntax and semantics of both kinds of languages within a single natural and mathematically precise theory.

Importantly, Montague uses only standard, well-understood tools from mathematics, and the formalism is "natural" in the sense of being *compositional*: the interpretation of a sentence is built from the interpretation of the words in it, as well as the syntactic rules used to combine them. This is important from the contemporary perspective of natural language processing applications as it is required for the *automatic* generation of interpretations, as well as being "natural" in the sense of mirroring our ability to comprehend a sentence by the combination of its components.

The *dynamic turn* in natural language semantics, attributed to Heim [1982] and Kamp [1981], relocated the meaning of a sentence from the logical form itself to its *context change potential*, interpreting new sentences in the context of those preceding. This enabled the interpretation of context-dependent meaning such as the referent of a pronoun, which had eluded Montague semantics.

Conversely, certain features of language handled well by Montague semantics – such as quantification and coordination phenomena – received less successful accounts in these dynamic semantics, which are not compositional. This resulted in the development of frameworks combining the twos approaches, such as the Dynamic Predicate Logic (DPL) of Groenendijk and Stokhof [1991] and the Compositional Dynamic Representation Theory (CDRT) of Muskens [1996], the latter notable for being both easy to use and "mathematically clean" – mathematically rigorous and using only standard type theory.

A more recent approach is the dynamic semantics of de Groote [2006], as extended by Lebedeva [2012], which goes beyond Montague semantics by incorporating *continuations* from programming language semantics (Strachey and Wadsworth [1974]) for a second notion of context as *the future of the discourse*. The result is a dynamic semantics in the style of Montague that firmly separates the context from the content of a sentence, while also preserving the properties of compositionality and the use of only common mathematical tools. Unlike DPL and CDRT, it does not rely on the use of assignment functions as semantic objects, and so avoids the *destructive assignment problem* whereby the existing value of a variable is lost when a new value is assigned. This necessitates the careful choice of variable names to correctly capture anaphora and also poses problems in accounting for modal subordination (see Asher and Pogodalla [2010a]). With these advantages, the extent to which de Groote's semantics can be used is an active area of research, with recent extensions to capture modal subordination (Asher and Pogodalla [2010a], Qian et al. [2016]); discourse structure (Asher and Pogodalla [2010b]); events (Qian and Amblard [2011]); verb phrase anaphora (Itegulov and Lebedeva [2018]); and attitude verbs and verbs of report (Bernard [2018]), as well as being implemented (Itegulov et al. [2018]).

This thesis continues the extension of de Groote's framework, in particular exploiting the flexibility of context by considering not just interaction with the context but interaction *within* the context to locate implicatures. Implicatures are situated in a group of meaning classes characterized by existing outside the plain semantic content of an utterance. Also in this group is presupposition – meaning assumed by an utterance for it to be meaningful – as in 'John quit smoking', which relies on John having smoked to make sense. If this presupposed information is not in the discourse context, it is accommodated alongside the plain content of the sentence. Implicature refers to meaning outside of what is explicitly said, logically entailed or presupposed by an utterance. It is traced back to Frege [1879] and was brought to prominence by Grice [1975]'s treatment that introduced a provisional division – with prevailing terminology – between *conversational implicature*, governed by principles of cooperative conversation such as utterances being relevant to what has come before, and *conventional implicature*, instead associated with particular words – *but*, for example, is said to implicate a contrast between two clauses, while not explicitly stating this contrast.

1.2 Thesis Contributions

If these meaning classes and their distinctions seem murky, it is because they are. A survey of these phenomena by Potts [2015] argues that their definitions are "still hotly contested" and suggests the refocus towards developing "rich theories of properties" that motivates this thesis. Lebedeva's extension of de Groote's framework goes some way towards this by accounting for presuppositions of referring expression, and proposes a mechanism for handling conversational implicatures. Treatment within the same framework allows a preliminary formal distinction between presuppositions and certain kinds of conversational implicatures to be made.

This thesis goes further by distilling Lebedeva's approach of *conversational implicatures by proof-theoretic abduction* into *implicatures by reasoning in the context*. By elaborating the context structure to a logical theory using the classical logic framework for reasoning as theory formation of Poole [1988, 1989, 1990], meaning associated with both conversational and conventional implicatures is captured while preserving the features of compositionality and the use of standard logical tools.

After showing how this approach works on the kinds of conversational implicatures considered in [Lebedeva, 2012, Section 7.2], it is applied to conventional implicatures via canonical instances of its two main characterizations: *but* as a traditional Gricean conventional implicature and supplements as examples of 'CIs' from Potts [2005b]. The result is a formal account of their meaning in a way that is closely related to conversational implicatures. In the case of *but*, it suggests a pragmatic theory as being the felicitous choice of connective for introducing meaning that contradicts an implicature of the context in which it is being introduced, as opposed to the lexical item explicitly encoding a contrast. For supplements, it formalizes the intuition that CIs provide "a clue as to how the [non-supplementary] information should be received" [Potts, 2005b, p.7] as coming from the interaction between supplementary and regular content.

With a treatment of some presuppositions, conversational implicatures and conventional implicatures with the same formalism, formal comparisons can begin to be made. To this end, definitions are proposed for properties associated with these meaning classes. This work is, of course, limited in scope with the number of examples it treats and so is not advocating a unified theory of implicatures – rather, showing how this kind of analysis can draw connections within these related meaning classes. Its merit is in providing many avenues to explore and promising analyses that connect meaning.

1.3 Thesis Structure

Chapter 2 discusses the relevant terminology and methodology from the formal semantics of natural language and presents the details of the formalisms on which this thesis is built: Montague semantics, de Groote [2006]'s framework G_0 and Lebedeva [2012]'s extension $GL\chi$. Chapter 3 surveys formal treatments of conversational implicatures, focusing on the approach by proof-theoretic abduction given as a further direction in Lebedeva [2012]. Justification is made for refactoring this method as implicatures by common-sense reasoning in the context; the new structure of context is detailed and applied to examples of conversational implicatures. The treatment of presuppositions in $GL\chi$ is revisited with the new context structure. Chapter 4 establishes the problem of capturing conventional implicatures with reference to current approaches and applies the method from the previous chapter to instances of *but* and supplements. Chapter 5 discusses the conclusions and contributions of this thesis, proposing formal definitions for properties of the meaning classes and future work.

Chapter 2

Formal Semantics of Natural Language

This is a thesis in the formal semantics of natural language. There are two components – natural language semantics from linguistics and formal semantics from logic. This chapter provides the relevant background in each and in both: logic, language and the current state of their intersection. After introducing the language used to talk about language and linguistic methodology, a descriptive and technical account of Montague semantics – the cornerstone of formal semantics for natural language – is given. It is then related to the semantics of programming languages via the notion of continuations. The extension of Montague semantics by de Groote [2006] is described, before introducing fully-formally the continuation-based dynamic semantics with exception handling in Lebedeva [2012]. The ability to capture the dynamic meaning class of presuppositions is demonstrated, to show the utility of – and familiarize the reader with – the framework.

2.1 Terminology and Methodology

Semantic inquiry of language often proceeds by way of a sentence posed for discussion. For example, (1) is used by Frege [1892/1948] to illustrate a distinction between sense (meaning) and reference (denotation) – the meaning of 'the morning star' is different to 'the evening star', although they both refer to Venus.

(1) The morning star is the evening star.

Such an example is often considered in the abstract, removed from how sentences are encountered in the wild – surrounded by other sentences in text or conversation.

The *dynamic turn* in semantics, originating with Heim [1982] and Kamp [1981], is specifically concerned with sentences in context. It recognizes that communication relies on having considerable information in common and goes beyond the surface meaning of a sentence by considering how interaction with external information bears on its interpretation. To make this distinction, example sentences will be called utterances in a discourse, and reference will be made to the participants of the discourse – *speaker* and *hearer*.[1] The informal notion of the *context of a discourse* refers to the participants' respective knowledge bases, comprised of common or background knowledge and the content from the preceding discourse.

This section situates semantics in linguistic theory alongside syntax and pragmatics and establishes criteria for assessing the usefulness of a semantic formalism.

2.1.1 Semantics, Syntax, and Pragmatics

Within linguistics, semantics is neighboured by *syntax* and *pragmatics*. Both are indispensable to formal semantics, the former because of the property of *compositionality*, whereby the meaning of a sentence is built from the meaning of its components and the syntactic rules by which they are combined, and the later because dynamic semantics is increasingly concerned with phenomena traditionally labelled pragmatic. For these reasons, many of the open problems in formal semantics occur at their boundaries, and any semantic formalism needs to interface with formalisms in these fields.

The syntax of natural language refers to the rules governing the structure of a sentence, capturing intuitions like the grammaticality of "Jayne plays drums" and the ungrammaticality of "Drums Jayne plays". In comparison, semantics is concerned with differences in meaning that cannot be accounted for by grammar, as in the ambiguity of "Someone plays every instrument", compared to the unambiguous meaning of "Jayne plays every instrument" – despite their similar structures. The relative ease in pinning down the character of syntax means it received formal treatments before semantics, and so there exist rich syntactic theories independently of semantics. For formal semantics, Partee [1973] identifies two criteria for an adequate syntactic theory, "(i) that they define the set of wffs [well-formed formulas] of the language... and (ii) most importantly, that they provide a basis for the rules of semantic interpretation." This is also the extent of our concern with syntax.

Pragmatics is concerned with language usage, considering meaning in context.

[1] There is research focusing on multiparty discourses. For our purposes, discourse is assumed to have two participants.

Its cornerstone is the conversational maxims of Grice [1975], which set out the otherwise unwritten rules of cooperative conversation, such as being as informative as required, but no more informative than necessary. Another key pragmatic concept, also considered by Grice, is *relevance* – the idea that successive utterances relate to each other in a meaningful way. Apparent violations of the conversational maxims and relevance in conversation lead to *implicatures* – additional meaning that allows the maxims to be satisfied. Rather than being concerned with the truth-conditions of a sentence, pragmatics is interested in the *felicitousness* of an utterance – whether it is acceptable to say a certain (grammatically well-formed) sentence in a particular context.

Dynamic semantics and pragmatics have been introduced thus far in strikingly similar terms, both characterized by a focus on context. As the more classically semantic problems that originally motivated dynamic semantics have been solved, such as discourse reference in presuppositions and anaphora, attention has turned to more traditionally pragmatic problems, like implicature. At the same time, pragmatics has incorporated more formal methods, resulting in the area of *formal pragmatics*[2]. In this way, they may be distinguished by tradition – pragmatics being rooted in the less formal tradition of Grice, and dynamic semantics coming from the formal origins of Heim and Kamp. Potts [2009] relates them with their common goal of *utterance interpretation*, as opposed to sentence interpretation:

> Utterance interpretation involves complex interactions among (i) semantic content, (ii) the context of utterance, and (iii) general pragmatic pressures (of which Grice's maxims are one conception). The starting point for formal pragmatics is the observation that speakers agree to a remarkable extent on the interpretations of the utterances they hear, suggesting that there are deep regularities across speakers, utterance contexts and sentence types in how (i)-(iii) interact.

Finally, a note on terminology. The syntax and semantics of natural language will often be referred to as just 'syntax' and 'semantics', where the context 'natural language' is clear. In formalizing the syntax and semantics, however, logical languages are used, for which we also speak of their syntax and semantics. In the context of a logical language, syntax refers to the symbols or purely formal expressions, and semantics refers a means of assigning meaning to the symbols, for example in a model. *Interpretation*, by default, is used to refer to the logical expression assigned to a lexical

[2] For a survey, see Potts [2009].

item or expression and not the usual usage in logic of interpretation of a semantics, for example, in a model.

2.1.2 Assessing the Usefulness of a Formal Semantics

Adopting Montague's premise, that there is "no important theoretical difference between natural languages and the artificial languages of logicians", inherits both the advantages and limitations of formal methods: just as an uninsightful logic may be defined, so too can an unilluminating natural language semantics, and so a means of assessing the usefulness of a particular semantics is required:

> ...it is actually difficult from our vantage point concerning mathematical modeling to make the claim that one or another model is incorrect; the strongest assertions we can make are that the results of the modeling are empirically false, or that they conflict with what "everyone in the field knows," or that they lead to uninteresting questions. (Moss [2011])

To address this, we mimic the practice from formal methods of providing a *specification* with respect to which a program may be proved correct, by identifying criteria to judge our formalism against.

Certain desired properties can be specified formally. Foremost is the compositionality of a semantics, desired because it enables the *automatic* translation of expressions into a logical form. Another property is that of being *meaning preserving*: if interpretations $[\![A]\!]$ and $[\![B]\!]$ of sentences A and B have the same logical form, then A and B have the same meaning. This is not purely formal, however, since determining whether two sentences have the same meaning is determined by intuition about language.

By the nature of the problem, most of the specification is informal. Dowty et al. [1981] identify three properties of a useful formalism. The first is the ability to account for puzzles by capturing parts of language that have not been satisfactorily formally captured, or understood, before. The second is the provision of principled explanations: "in semantics, just as in syntax, we require our theory to provide *principled* explanations for the facts, i.e., explanations that emerge from a tightly interconnected system of general statements and which lead to further predictions about as yet undiscovered facts" (Dowty et al. [1981]). Finally, it must join "in a plausible way" with theories in related areas, such as theories of syntax. This is related to the *modularity* of a formalism.

Combined with earlier remarks from Moss, we are left with four criteria:

1. Accounts for puzzles
2. Provides principled explanations
3. Joins with theories in related areas
4. Leads to interesting questions

We will return to these in the concluding chapter to assess the usefulness of the formalism developed.

An additional consideration is the motivation for the natural language semantics. Potts [2009] observes that formal pragmatics tends to extend existing semantic theories, and claims that this leads to the bias of "an emphasis on interpretation (over production) and a tendency to try to find single, fixed solutions". Design choices vary greatly depending on the motivation for the framework. To go some way to addressing these biases, we identify the motivations of this thesis as dual – the automatic interpretation of discourse in a logical form, and the use of logical interpretations to understand natural language. As the formalism is developed, instances where these goals compete are addressed.

2.2 Montague Semantics

Montague semantics (Montague [1970a,b, 1973]) is the foundation of contemporary formal semantics of natural language. Prior to Montague, the prevailing view was that natural language meaning was too unsystematic to be formalized in an insightful way, or to be formalized altogether: "If we were to devise a logic of ordinary language for direct use on sentences as they come, we would have to complicate our rules of inference in sundry unilluminating ways" (Quine [1960]). Partee [2005] explains Montague's innovations:

> Two aspects of Montague's approach looked especially exciting. The first was the then-revolutionary (to a linguist) idea that the core data to be accounted for were the truth conditions of sentences, and semantic values of other constituents should be worked out so as to compositionally combine to give the right truth conditions for the whole sentences. Suddenly there was a non-subjective criterion of "observational adequacy" for semantics, where there had been none before.

> The second exciting aspect of Montague's approach was that it incorporated some powerful tools that would let semantics do some real work, which in turn could help keep the syntax clean and elegant.

Natural language semantics has diversified since, but Montague semantics remains either a starting point or reference point for the majority of semantic theories – the work built on in this thesis is 'Towards a Montagovian Account of Dynamics' (de Groote [2006]). To clarify the sense in which our approach is and is not Montagovian, three main properties are presented here, as identified by Dowty et al. [1981].

Truth-conditional The first exciting idea to which Partee refers, truth-conditional semantics, originates with Tarski [1935] and identifies the meaning of a sentence with its truth conditions, where 'true' means 'corresponds to the way the world is'. For example, a truth-conditional account of "Dickson is north of Ainslie" specifies the entities Dickson and Ainslie, as well as the relation "north of", such as spatial and temporal features. This distinguishes an *object language* and a *meta-language* used to talk about the sentences in the object language. In this case, English is used for both but this is not necessary – for example, a non-linguistic meta-language could be used. Truth-conditional semantics is the realm of *compositionality*.

Model-theoretic A model formally specifies the entities in a world, with respect to which an *interpretation* of the object language can be made. As such, model-theoretic semantics can be thought of as "a method... for carrying out the program of truth-conditional semantics" (Dowty et al. [1981]). Certain words have the same interpretation in every model – such as *not*, *and*, *every* and *some*. For example, in the sentence "Everyone loves someone", the ambiguity is associated with *someone* and *everyone*, not *loves*, and so a theory should give the same account of ambiguity for this sentence as it does for "Everyone hates someone". Entailment of meaning can be accounted for within model theory as well.

Possible world-based Truth-conditions are how the world would have to be for a particular arrangement of entities and relations to attain in that world. Reference may be made not only to this world by considering truth relative to other possible worlds. This is the idea of possible world semantics, capable of capturing notions like possibility and necessity.

The emphasis in de Groote [2006] and Lebedeva [2012], on which this thesis builds, are the two exciting aspects to which Partee – compositionality, from truth-conditional semantics, and the use of familiar tools from mathematical logic. The use of model-theoretic and possible world semantics for interpreting the logical form given by the framework is compatible but optional.

Before presenting a formal account, the scope of Montague semantics is clarified. Dowty et al. [1981] extract the following aspects of natural language semantics with which Montague semantics is not concerned.

Lexical semantics The meaning of words and their relations, such as synonymy and antonymy, is a separate field. A compositional semantics has the advantage of being able to easily accommodate treatments of lexical semantics, since it is concerned with building meaning out of components, but itself is not concerned with the analysis of "atomic" expressions.

Non-declarative sentences Montague semantics is only concerned with declarative sentences, as opposed to questions, given that it comes from mathematical logic, which is exclusively statements. This is the case for other features of natural language, such as tense, which are treated as distinct problems to incorporate into a semantics and are not the concern of this thesis.

Psychological reality Although there is a process of interpretation being defined, no claim is made that Montague semantics represents cognition – how meaning is psychologically processed.

We proceed by presenting a Montague-style semantics, in line with de Groote [2006] and Lebedeva [2012]. Rather than the exact system described by Montague, it is a simpler, modern presentation that nonetheless preserves the features of the original.

2.2.1 Syntax

To provide a compositional semantics requires specifying a syntax, and this is exactly how concerned Montague semantics is with syntax. Montague used an ad-hoc syntactic system in the style of a *categorial grammar* for the sake of achieving compositionality. Categorial grammar refers to a family of formalisms that capture natural language syntax by specifying a set of syntactic categories and inference rules governing how they interact, as well as a *lexicon* – a function that maps words to syntactic categories. Not committing to a precise syntactic theory also allows the approach to be used with different syntactic formalisms.

To specify a categorial grammar $\mathcal{G}$, the following definitions are made.

Definition 2.1 (Syntactic categories)**.** The set Syn of syntactic categories in the grammar $\mathcal{G}$ is defined by the following formal grammar:

$$\text{Syn} := n \mid np \mid S \mid \text{Syn}/\text{Syn} \mid \text{Syn}\backslash\text{Syn} \tag{2.1}$$

The members of Syn are thought of as different grammatical classes – n corresponds to nouns, np to noun phrases and S to sentences. Some of the complex members of Syn also correspond to named grammatical classes, for example, np/n corresponds to determiners, n/n to adjectives and $(np\backslash s)/np$ to transitive verbs. Expressions a/b and $a\backslash b$ are thought of like functions – they are both categories that take a member of category b and return a member of category a, but $b\backslash a$ takes a word on the *left* rather than the right. This is captured by the following inference rules.

Definition 2.2 (Inference rules)**.** For $u, v \in$ Syn, the inference rules in the grammar $\mathcal{G}$ are:

$$\frac{u(u\backslash v)}{v}\ \backslash_e$$

$$\frac{(v/u)u}{v}\ /_e$$

An example of a tiny lexicon $\mathcal{L}$ in the grammar $\mathcal{G}$ is given in Table 2.1 by specifying the syntactic categeories of some words.

Word	Syntactic category
Jayne	np
drums	n
plays	$(np\backslash S)/np$

Table 2.1: Tiny lexicon $\mathcal{L}$.

A *language* can then be generated by combining the lexicon with the inference rules. By representing the lexicon with judgements of the following form

$$\langle\text{word}\rangle : \langle\text{syntactic category}\rangle \tag{2.2}$$

members of the language are associated with deduction proofs, as illustrated by the following example.

Example 2.3 (*Jayne plays drums* syntax)**.** *Jayne plays drums* is a sentence in the language generated by the lexicon $\mathcal{L}$, as evidenced by the following proof.

$$\dfrac{\text{Jayne} : np \qquad \dfrac{\text{plays} : (np\backslash S)/np \qquad \text{drums} : np}{\text{plays drums} : np\backslash S}\,/_e}{\text{Jayne plays drums} : S}\,\backslash_e$$

The sentence "drums Jayne plays" does not belong to the language generated by the lexicon.

The judgement form (2.2) is intentionally suggestive of type theory – the syntax-semantics interface in this formalism depends on the Curry-Howard correspondence. Before making this clear, the basic semantics is defined. Th

2.2.2 Semantics

Montague semantics is an extension of Church's simply typed λ-calculus. It uses the following types.

Definition 2.4 (Semantic types)**.** The set Sem of types is defined by the following abstract grammar:

$$\text{Sem} := \iota \mid o \mid \text{Sem} \to \text{Sem}$$

Terms of type ι are thought of as *entities* and terms of type o as *propositions*. There is a map from syntactic types to semantic types providing the semantic interpretation.

Definition 2.5 (Semantic interpretation)**.** Let M be the interpretation function from syntactic categories to semantic types, replacing the two connectives of the syntactic grammar with intuitionistic $\to$:

$$\begin{aligned} M : \text{Syn} &\to \text{Sem} \\ s &\mapsto o \\ np &\mapsto \iota \\ n &\mapsto \iota \to o \\ \text{Syn}/\text{Syn} &\mapsto M(\text{Syn}) \to M(\text{Syn}) \\ \text{Syn}\backslash\text{Syn} &\mapsto M(\text{Syn}) \to M(\text{Syn}) \end{aligned}$$

Under this semantic interpretation, nouns are subsets of the set of entities, noun phrases are entities, and sentences are propositions. Intransitive verbs such as *sings* and nouns such as *chicken* have type $\iota \to o$, thought of as the set of entities who sing or that are chickens. Intransitive verbs such as *loves* have semantic type $\iota \to \iota \to o$, thought of as pairs of entities such that the first entity loves the second entity.

For a member of the lexicon w with syntactic type t, there is a corresponding λ-term τ with type $M(t)$. To illustrate, the syntactic types in Example 2.3 are replaced by the semantic types in the following proof:

$$\dfrac{\dfrac{}{[\![Jayne]\!] : \iota} \qquad \dfrac{[\![plays]\!] : \iota \to \iota \to o \qquad [\![drums]\!] : \iota}{[\![plays]\!]\, [\![drums]\!] : \iota \to o} \to_E}{([\![plays]\!]\, [\![drums]\!])[\![Jayne]\!] : o} \to_E$$

By the Curry-Howard correspondence, this proof can be viewed as a simply typed λ-term. This is the syntax-semantics interface, connecting categorial grammar with Montague semantics. The meaning of the sentence can now be computed compositionally from this form and λ-terms for the individual lexical items. Suppose the logic has constants $\mathbf{d} : \iota, \mathbf{j} : \iota$ and $\mathbf{play} : \iota \to \iota \to o$. Then we assign the following terms as the semantic interpretations of *Jayne*, *plays* and *drums*:

$$\begin{aligned} [\![Jayne]\!] &= \mathbf{j} \\ [\![drums]\!] &= \mathbf{d} \\ [\![plays]\!] &= \lambda xy.\mathbf{play}\ y\ x \end{aligned}$$

The semantic interpretation of the sentence is built out of the semantic interpretations of the components, as well as the syntactic types that determine how they are put together. This is β-reduced to a normal form:

$$\begin{aligned} ([\![plays]\!]\, [\![drums]\!])[\![Jayne]\!] &= \Big((\lambda xy.\mathbf{play}\ y\ x)\ \mathbf{d}\Big)\ \mathbf{j} \\ &\to_\beta (\lambda y.\mathbf{play}\ y\ \mathbf{d})\ \mathbf{j} \\ &\to_\beta \mathbf{play}\ \mathbf{j}\ \mathbf{d} \end{aligned}$$

The sentence is now associated with an expression in a logical language, found compositionally from the logical forms associated with its components. This is referred to as the *interpretation* of the sentence. It may be treated like any other logical expression and be given an interpretation – in the sense usually used in logic – for example, by interpretation in a model. Problems that exist at the natural language level, such as the truth or falsity of a sentence and whether one sentence entails another, have easy solutions at the level of semantic interpretation of a logical form. Certain natural language semantics skip the intermediate step of *indirect translation* in a logic language, instead providing a *direct translation* into, say, a model. The advantage of an indirect translation is the flexibility of the final interpretation – all of the tools of logic for interpreting a formal expression are at one's disposal.

2.2.3 Type-raising for Quantified Noun Phrases and Scope Ambiguity

The last example was straightforward. To support the contention that natural language may be given an insightful formal translation, Montague contributed interpretations of more difficult features of language, in particular, quantifed noun phrases and scope ambiguities, by *type-raising* – identifying an entity with the set of its properties. Quantified noun phrases such as *everybody*, *somebody* and *nobody* take the place of regular noun phrases, as in the following examples:

(5) Everybody plays drums.

(6) Everybody loves somebody.

However, it does not make sense to capture them as constants with type ι of entities since this ignores the sense in which they quantify. Instead, they are given the following interpretations:

$$\begin{aligned} [\![\mathit{everybody}]\!] &= \lambda P.\forall x.Px : (\iota \to o) \to o \\ [\![\mathit{somebody}]\!] &= \lambda P.\exists x.Px : (\iota \to o) \to o \\ [\![\mathit{nobody}]\!] &= \lambda P.\neg\exists x.Px : (\iota \to o) \to o \end{aligned}$$

The type $\iota \to o$ is thought of as the set of entities satisfying a particular property, so $(\iota \to o) \to o$ is a set of properties of individuals.

To illustrate the need for type-raising, consider the desired interpretation of (5):

$$\forall x.\mathbf{play}\; x\; \mathbf{d}$$

From the terms so far assigned to nouns and verbs, its interpretation is computed compositionally as follows:

$$
\begin{aligned}
(\llbracket plays \rrbracket\,\llbracket drums \rrbracket\,)\llbracket Everybody \rrbracket \;&= \Big(\left(\lambda xy.\mathbf{play}\;\; y\; x\right)\mathbf{d}\Big)\left(\lambda P.\forall x.Px\right)\\
&\rightarrow_\beta \left(\lambda y.\mathbf{play}\;\; y\;\mathbf{d}\,\right)\left(\lambda P.\forall x.Px\right)\\
&\rightarrow_\beta \mathbf{play}\;\;\left(\lambda P.\forall x.Px\right)\;\mathbf{d}
\end{aligned}
$$

To get the desired interpretation requires raising the types of $\llbracket plays \rrbracket$ and $\llbracket drums \rrbracket$ as follows:

$$
\begin{aligned}
\llbracket drums \rrbracket \;&= \lambda P.P\mathbf{d}\;:\left(\iota \rightarrow o\right)\rightarrow o\\
\llbracket plays \rrbracket \;&= \lambda XY.Y\Big(\lambda x.X\left(\lambda y.\mathbf{play}\;\; x\; y\right)\Big):\big(\left(\iota \rightarrow o\right)\rightarrow o\big)\rightarrow\big(\left(\iota \rightarrow o\right)\rightarrow o\big)\rightarrow o
\end{aligned}
$$

To justify this move, type-raised interpretations are thought of as depending on a set of individuals for which a particular property holds, and an individual as the set of all properties of the individual. Then the interpretation of ((5)) is:

$$
\begin{aligned}
(\llbracket plays \rrbracket\,\llbracket drums \rrbracket\,)\llbracket Everybody \rrbracket \;&= \Bigg(\Bigg(\lambda YX.X\Big(\lambda x.Y\left(\lambda y.\mathbf{play}\;\; x\; y\right)\Big)\Bigg)\left(\lambda P.P\mathbf{d}\,\right)\Bigg)\left(\lambda P.\forall x.Px\right)\\
&\rightarrow_\beta \Bigg(\lambda X.X\Big(\lambda x.\left(\lambda P.P\mathbf{d}\,\right)\left(\lambda y.\mathbf{play}\;\; x\; y\right)\Big)\Bigg)\left(\lambda P.\forall x.Px\right)\\
&\rightarrow_\beta \Bigg(\lambda X.X\Bigg(\lambda x.\Big(\left(\lambda y.\mathbf{play}\;\; x\; y\right)\mathbf{d}\Big)\Bigg)\Bigg)\left(\lambda P.\forall x.Px\right)\\
&\rightarrow_\beta \Bigg(\lambda X.X\Big(\lambda x.\left(\mathbf{play}\;\; x\;\mathbf{d}\,\right)\Big)\Bigg)\left(\lambda P.\forall x.Px\right)\\
&\rightarrow_\beta \left(\lambda P.\forall x.Px\right)\Big(\lambda x.\left(\mathbf{play}\;\; x\;\mathbf{d}\,\right)\Big)\\
&\rightarrow_\beta \forall x.\Big(\lambda x.\left(\mathbf{play}\;\; x\;\mathbf{d}\,\right)x\\
&\rightarrow_\beta \forall x.\mathbf{play}\;\; x\;\mathbf{d}
\end{aligned}
$$

The technique of type-raising also captures *scope ambiguity*, as illustrated in (6). Scope ambiguity is where the scope is not determined by the syntax – it is wider than the apparent syntactic scope. The two different scope interpretations of (6), called *subject wide scope* and *object wide scope*, can be specified by the interpretation of the verb as one of the following:[3]

$$[\![loves]\!]_s = \lambda OS.S(\lambda x.O(\lambda y.\textbf{love}\,xy))$$
$$[\![loves]\!]_o = \lambda OS.O(\lambda y.S(\lambda x.\textbf{love}\,xy))$$

2.3 De Groote and Lebedeva's Continuation-based Dynamic Semantics

Dynamic semantics continues the extension of natural language phenomena that can be captured formally. Other prominent dynamic semantic theories, as surveyed in [Lebedeva, 2012, Section 2.2], deviate in some way from two principles of Montague semantics: compositionality and the use of standard tools from logic. The loss of compositionality means an interpretation cannot be *automatically* generated, and the use of ad-hoc definitions or non-standard tools limits accessibility and sacrifices insights gained from applying well-understood formalisms.

The framework of de Groote [2006] returns to these principles. By adding a third atomic type to the λ-calculus, a notion of context is incorporated in a compositional way. The innovation is the use of a technique from programming language semantics called *continuation-passing style* (Strachey and Wadsworth [1974]). After introducing continuations and relating them to type-raising in Montague semantics, their use by de Groote is described. The formal details of Lebedeva [2012]'s framework GL – a systematic translation from Montague's static interpretation to dynamic interpretations in the style of de Groote – is then presented, followed by Lebedeva [2012]'s framework $GL\chi$, which incorporates an exception raising and handling mechanism to capture presuppositions.

2.3.1 Continuations in Natural Language Semantics

Dynamic phenomena in natural language are characterized by dependence on context and pose a challenge for compositionality. Context-dependent features are also found in programming languages, such as *goto statements*, meaning the problem of providing compositional semantics for dynamic phenomena has occurred in programming languages as well. This is the origin of continuation-passing style, used to formalize goto statements by Strachey and Wadsworth [1974].

[3] See Lebedeva [2012], computations (1.10) and (1.11), for complete reductions of subject wide and object wide scope interpretations for (6).

To see what a continuation is in the original context of programming languages, consider the original set-theoretic semantics from Strachey and Wadsworth [1974]. Let State be the set of *states* of a system and θ be a *state transformation*, that is, a function $\theta : \mathrm{State} \to \mathrm{State}$. A state transformation results from executing a *command* $\gamma \in \mathrm{Cmd}$ with respect to an *environment* $\rho \in \mathrm{Env}$, which provides the denotations for the *identifiers* occuring in the command γ. We can now define a semantic function $\mathcal{C} : \mathrm{Cmd} \to (\mathrm{Env} \to (\mathrm{State} \to \mathrm{State}))$, with the value of the command γ given as follows:

$$\mathcal{C}[\![\gamma]\!](\rho) = \theta$$

It is natural to then interpret the sequencing of commands $\gamma_0; \gamma_1$ as performing one state transformation after another, as in:

$$\begin{aligned} \mathcal{C}[\![\gamma_0; \gamma_1]\!](\rho) &= \mathcal{C}[\![\gamma_1]\!](\rho) \circ \mathcal{C}[\![\gamma_0]\!](\rho) \\ &= \theta_1 \circ \theta_0 \end{aligned}$$

However, if γ_0 contains a jump to another command, then γ_1 will not be performed and these semantics do not capture the meaning of the program.

The idea of continuations is to define a semantic function involving not the state transformation for a command in isolation, but the state transformation that would be produced by a command *from this point to the end of the program*. This requires adding another argument $\phi \in \mathrm{State} \to \mathrm{State}$ that is the state transformation specified by the rest of the program and called a continuation. The new semantic function is $\mathcal{P} : \mathrm{Cmd} \to (\mathrm{Env} \to ((\mathrm{State} \to \mathrm{State}) \to (\mathrm{State} \to \mathrm{State})))$ and the sequencing of commands becomes:

$$\mathcal{P}[\![\gamma_0; \gamma_1]\!]\rho\phi = \mathcal{P}[\![\gamma_0]\!]\rho\{\mathcal{P}[\![\gamma_1]\!]\rho\phi\} \tag{2.3}$$

Continuation-passing style (CPS) is a way of writing a function with an extra argument making the "continuation" or future of the computation explicit. Plotkin [1975]'s call by value CPS translation in λ-calculus is as follows:

$$\begin{aligned} \overline{x} &= \lambda\kappa.\kappa x \\ \overline{\lambda x.M} &= \lambda\kappa.\kappa(\lambda x.\overline{M}) \\ \overline{MN} &= \lambda\kappa.\overline{M}(\lambda m.\overline{N}(\lambda n.mn\kappa)) \end{aligned}$$

Continuations have been used in different ways in natural language semantics since the work of Barker [2001] and de Groote [2001], the latter making the observation that Montague's type-raising is, in essence, a CPS transformation. This is illustrated by comparing Table 2.2 with the CPS transformation above.

Lexical item	Standard interpretation	Type-raised interpretation
Mary	m	$\lambda\kappa.\kappa\mathrm{m}$
John	j	$\lambda\kappa.\kappa\mathrm{j}$
loves	$\lambda x.\lambda y.\mathbf{love}\,y\,x$	$\lambda f.\lambda g.f(\lambda x.g(\lambda y.\mathbf{love}\,y\,x))$

Table 2.2: Comparison of direct and Montague interpretations.

In de Groote [2006], the new type γ of contexts represents a *left context*, that made of the preceding sentences, and a *right context*, made of the discourse to come. This second context is a discourse to be interpreted as a proposition, provided it is given a left context: that is, it has type $\gamma \to o$. A sentence takes a left context and a right context and returns a proposition, so the interpretation of a sentence is of type $\gamma \to (\gamma \to o) \to o$. Then the sequencing of sentences is given by:

$$[\![S_1; S_2]\!] = \lambda e\phi.[\![S_1]\!]e(\lambda e'.[\![S_2]\!]e'\phi) \tag{2.4}$$

The first sentence S_1 is passed a left context that is the same as the left context of $(S_1; S_2)$ and a right context that is the right context of $(S_1; S_2)$ plus the right context of S_2. Changing the notation in (2.3) so that double brackets are the semantic function gives an expression similar to (2.4):

$$[\![\gamma_0; \gamma_1]\!] = \lambda\rho\phi.[\![\gamma_0]\!]\rho\{[\![\gamma_1]\!]\rho\phi\}$$

With de Groote [2006]'s innovation, we proceed by presenting the formalism GL and its extension $GL\chi$, illustrating with long-running examples based on (53) and (55) from [Lebedeva, 2012, p.207], relabelled (7) and (8) respectively:

(7) If John is back, then his child is happy.

(8) If John has a child, then his child is happy.

These pieces will be put together in Section 2.3.4.2 to demonstrate how the problem of presupposition projection is accounted for in $GL\chi$.

2.3.2 Framework GL

Lebedeva [2012] uses de Groote [2006]'s framework G as a basis for framework GL, which achieves more compact dynamic representations that are closer analogues to the original static formulations. We begin by defining the terms and types of the language.

Definition 2.6 (GL terms)**.** Given an enumerable set of variables $\{x, x_1, x_2, \ldots, \}$, an enumerable set of primitive constants $\{c, c_1, c_2, \ldots, \}$, logical constants $\exists, \wedge$ and $\neg$ and two special constants upd and sel, the set of terms in GL is defined by the following formal grammar:

$$t := x \mid c \mid \lambda x.t \mid tt \mid \exists t \mid t \wedge t \mid \neg t \mid \mathrm{upd}(t, t) \mid \mathrm{sel}\ t$$

The first four terms define a standard λ-calculus, with variables, constants, λ-abstraction and application. Adding the next three terms gives a standard Montague grammar, allowing terms that look like first-order logic propositions to be formed in the language. The other logical connectives are captured by introducing their symbols as abbreviations of the following:

$$\vee := \lambda \mathbf{AB}.\neg(\neg\mathbf{A} \wedge \neg\mathbf{B}) \tag{2.5a}$$

$$\rightarrow := \lambda \mathbf{AB}.\neg(\mathbf{A} \wedge \neg\mathbf{B}) \tag{2.5b}$$

$$\forall := \lambda \mathbf{A}.\neg(\exists(\neg\mathbf{A})) \tag{2.5c}$$

The final two terms are specific to de Groote-style dynamic semantics, for interaction between the content and the context of a discourse.

Definition 2.7 (GL types)**.** The set T of types in GL is defined by the following abstract grammar:

$$\mathrm{T} := \iota \mid o \mid \gamma \mid \mathrm{T} \rightarrow \mathrm{T}$$

The type ι is thought of as the type of *individuals*, o as the type of *propositions* and γ as the type of *left context*, made of the preceding sentences. Type $T \rightarrow T$ allows complex types to be built from these atomic types; of particular interest is the complex type $\gamma \rightarrow o$, interpreted as the *right context*, made of the sentences to follow. To make sense of this type – the type of continuations – consider that the upcoming sentences are interpreted as propositions (type o) once they receive a left context (type γ).

The semantic interpretation of these types is not rigid. The type of propositions may be interpreted extensionally as truth values or intensionally as a function from possible worlds to truth values, and the type of context is a list of names in de Groote [2006] and a conjunction of propositions in Lebedeva [2012]. This flexibility is an advantage of the framework.

Terms and types are related via typing judgements.

Definition 2.8 (*GL* typing judgements)**.** The judgement $t : \tau$, interpreted as t has type τ, is derivable from the basis Δ, given by the judgement $\Delta \vdash t : \tau$, if $\Delta \vdash t : \tau$ can be produced using the following rules for each kind of term:
Variables:

$$\frac{}{\Gamma, x : \alpha \vdash x : \alpha}$$

Abstraction:

$$\frac{\Gamma, x : \alpha \vdash v : \beta}{\Gamma \vdash \lambda x.v : \alpha \to \beta}$$

Application:

$$\frac{\Gamma \vdash v : \alpha \to \beta \qquad \Gamma \vdash u : \alpha}{\Gamma \vdash vu : \beta}$$

Logical constants:

$$\frac{}{\Gamma \vdash \top : o}$$

$$\frac{}{\Gamma \vdash \wedge : o \to o \to o}$$

$$\frac{}{\Gamma \vdash \exists : (\iota \to o) \to o}$$

$$\frac{}{\Gamma \vdash \neg : o \to o}$$

Special constants:

$$\frac{}{\Gamma \vdash \mathrm{sel} : (\iota \to o) \to \gamma \to o}$$

$$\frac{}{\Gamma \vdash \mathrm{upd} : o \to \gamma \to \gamma}$$

Linguistic constants:

$$\overline{\Gamma \vdash c_{iv} : \iota \to o}$$

$$\overline{\Gamma \vdash c_{tv} : \iota \to \iota \to o}$$

$$\overline{\Gamma \vdash c_{n} : \iota \to o}$$

$$\overline{\Gamma \vdash c_{np} : (\iota \to o) \to o}$$

$$\overline{\Gamma \vdash c_{rp} : (((\iota \to o) \to o) \to o) \to (\iota \to o) \to (\iota \to o)}$$

The linguistic constants $c_{iv}, c_{tv}, c_n, c_{np}$, and c_{rp} are intransitive verbs, transitive verbs, nouns, noun phrases and relative pronouns respectively. Their typing rules encode the grammatical rules by which they are governed.

2.3.2.1 Dynamization

The classical interpretation of a lexical item is static in the sense that it does not capture the dynamics of discourse – the interactions between the content and context. In de Groote [2006]'s framework, the basic type of propositions is dynamized by parametrization with a left context of type γ and a right context of type $\gamma \to o$. In addition to this, Lebedeva [2012] dynamizes the type of an individual by parametrizing it with respect to a left context, as in the following definition.

Definition 2.9 (Dynamization of types)**.** For atomic types ι and o, type parameter γ and any types α and β, dynamized types are:

$$\bar{\iota} := \gamma \to \iota \tag{2.6a}$$

$$\bar{o} := \gamma \to (\gamma \to o) \to o \tag{2.6b}$$

$$\overline{\alpha \to \beta} := \overline{\alpha} \to \overline{\beta} \tag{2.6c}$$

Lebedeva [2012] notes that a systematic translation of static interpretations to interpretations with a notion of context is straightforward for all varieties of terms except the linguistic constants. To enable a complete translation, [Lebedeva, 2012, Definition 4.27] gives the following mutually dependent functions that take static terms to dynamic terms and vice versa.

Definition 2.10 (Dynamization and reading functions)**.** The dynamization function $\mathbb{D}_\tau$ has type $((\gamma \to \tau) \to \overline{\tau})$ and is inductively defined on the type τ as follows:

$$\mathbb{D}_\iota[\mathsf{a}] := \mathsf{a} \tag{2.7a}$$

$$\mathbb{D}_o[\mathrm{P}] := \lambda e\phi.\mathrm{P}e \wedge \phi(\mathrm{upd}(\mathrm{P}e, e)) \tag{2.7b}$$

$$\mathbb{D}_{\alpha\to\beta}[f] := \lambda \mathbf{a}.\mathbb{D}_\beta[\lambda e.f\ e\ \mathbb{S}_\alpha[\mathbf{a}, e]] \tag{2.7c}$$

The reading function $\mathbb{S}_\tau$ has type $(\overline{\tau} \to \gamma \to \tau)$ and is inductively defined on the type τ as follows:

$$\mathbb{S}_\iota[\mathbf{a}, e] := \mathbf{a}e \tag{2.8a}$$

$$\mathbb{S}_o[\mathbf{P}, e] := \mathbf{P}e(\lambda e.\top) \tag{2.8b}$$

$$\mathbb{S}_{\alpha\to\beta}[\mathbf{f}, e] := \lambda a.\mathbb{S}_\beta[\mathbf{f}\ \mathbb{D}_\alpha[\lambda e.a], e] \tag{2.8c}$$

Equation (2.7a) says that dynamization of a term of type ι is trivial and equation (2.7c) specifies dynamization for complex types. The interesting part of the definition is equation (2.7b), which says that a dynamic proposition is a static proposition, parametrized by a left and right context, which updates the left context with the static proposition ($\mathrm{upd}(\mathrm{P}e, e)$) and passes this updated context to the continuation of the discourse. The reading function is given above to complete the definition of the dynamization function, however the details are not necessary for our purposes (see [Lebedeva, 2012, p.117] for an explanation). The convention of denoting a dynamic proposition, type $\gamma \to (\gamma \to o) \to o$, with boldface is used throughout.

With the dynamization function in hand, a complete systematic translation from static terms to dynamic terms may be defined.

Definition 2.11 (Dynamization of λ-terms)**.** Dynamization of a term t of type τ to give a term $\overline{t}$ of type $\overline{\tau}$ is defined recursively as follows:

$$\overline{x} = x \tag{2.9a}$$

$$\overline{\lambda x.u} = \lambda x.\overline{u} \tag{2.9b}$$

$$\overline{u(v)} = \overline{u}(\overline{v}) \tag{2.9c}$$

$$\overline{c} = \mathbb{D}_\tau[\lambda e.c] \tag{2.9d}$$

$$\overline{\exists u} = \overline{\exists}\,\overline{u} \qquad \text{where } \overline{\exists} := \lambda \mathbf{A}.\lambda e\phi.\exists(\lambda x.\mathbf{A}(\lambda e'.x)e\phi) \tag{2.9e}$$

$$\overline{u \wedge v} = \overline{u} \mathbin{\overline{\wedge}} \overline{u} \qquad \text{where } \overline{\wedge} := \lambda \mathbf{AB}.\lambda e\phi.\mathbf{A}e(\lambda e.\mathbf{B}e\phi) \tag{2.9f}$$

$$\overline{\neg u} = \overline{\neg}\overline{u} \qquad \text{where } \overline{\neg} := \lambda \mathbf{A}.\lambda e\phi.\neg(\mathbf{A}e(\lambda e.\top)) \wedge \phi e \tag{2.9g}$$

The dynamization function is used in (2.9d) for the linguistic constants. The dynamic conjunctions are defined in (2.9e)-(2.9g): dynamic existential quantifier introduces a new existentially quantified variable; dynamic conjunction places the second conjunct in the continuation of the first conjunct; and dynamic negation empties the context being passed to the continuation, so that variables introduced for binding within the scope of the negation are not available outside that scope.

The constants corresponding to the dynamizations of the other logical connectives are given by the following abbreviations, the dynamizations of the static abbreviations from (2.5):

$$\overline{\vee} = \lambda \mathbf{AB}.\overline{\neg}(\overline{\neg}\mathbf{A} \mathbin{\overline{\wedge}} \overline{\neg}\mathbf{B}) \tag{2.10a}$$

$$\overline{\Rightarrow} = \lambda \mathbf{AB}.\overline{\neg}(\mathbf{A} \mathbin{\overline{\wedge}} \overline{\neg}\mathbf{B}) \tag{2.10b}$$

$$\overline{\forall} = \lambda \mathbf{A}.\overline{\neg}(\overline{\exists}(\overline{\neg}\mathbf{A})) \tag{2.10c}$$

2.3.2.2 Examples

To see the framework in action, we proceed by computing the interpretations of (7) and (8). The examples are believed to be instructive at this point but include two details missing from the presentation thus far – the definitions of sel and upd, and the non-standard dynamization of complex lexical items.

Regarding the functions of context interaction, since sel is given only a partial definition by Lebedeva in GL – a full definition requiring the exception raising and handling mechanism of $GL\chi$ – we leave the details of upd and sel to the next section. For now, understand terms of the form sel P e as selecting an individual from context e satisfying the static property P, and $\text{upd}(\text{P}e, e)$ as updating context e with the proposition $\mathrm{P}e$. Sometimes it is necessary to select an individual from the context according to a *dynamic* property; for use in these instances, $\widetilde{\mathrm{sel}}$ is defined to have the same behaviour as sel except with respect to a dynamic property.

While the dynamic interpretations of many lexical items may be found by applying the dynamization function to their static interpretation, certain complex classes of lexical items containing extra semantic content are given different dynamizations. In the following examples, the proper name *John* does not have the dynamization according to Definition 2.11:

$$\overline{[\![John]\!]} = \lambda \mathbf{P}.\mathbf{P}(\lambda e.\mathbf{j})$$

This is not a desirable interpretation for a proper name because it requires that the logical language contains, a priori, as many constants as there are individuals to which natural language makes reference – an impractical requirement to be avoided. In addition, it does not agree with the analysis of proper names as referring expressions – lexical items containing descriptive content with which a referent may be retreived from the context.[4] The descriptive content of *John* is the property of being named "John" and is captured by the following dynamization:

$$\widetilde{[\![John]\!]} = \lambda \mathbf{P}.\mathbf{P}(\mathrm{sel}(\text{named "John"}))$$

From this point onwards, bespoke dynamic interpretations outside of Definition 2.11 are indicated with a tilde.

Proceding with the examples, tables 2.3 and 2.4 show the dynamic and static types and terms for the interpretation of the lexical items in utterances (7) and (8), illustrating how *GL*'s systematic dynamization achieves a dynamic term that is a close analogue of the static term. Note also that not all items may be given a static interpretation because their meaning is inherently context-dependent; such instances are indicated by a question mark.

[4]For a more detailed discussion of proper names and their treatment in the framework, see [Lebedeva, 2012, Section 5.2.2].

Lexical item	Syntax	Dynamic type	Dynamic interpretation
John	np	$((\bar{\iota} \to \bar{o}) \to \bar{o})$	$\lambda \mathbf{P}.\mathbf{P}(\text{sel}(\text{named "John"}))$
is_back	$np\backslash S$	$((\bar{\iota} \to \bar{o}) \to \bar{o}) \to \bar{o}$	$\lambda \mathbf{X}.\mathbf{X}(\lambda \mathbf{x}.\overline{\mathbf{back}}\,\mathbf{x})$
has	$(np\backslash S)/np$	$((\bar{\iota} \to \bar{o}) \to \bar{o}) \to ((\bar{\iota} \to \bar{o}) \to \bar{o}) \to \bar{o}$	$\lambda \mathbf{YX}.\mathbf{X}(\lambda \mathbf{x}.\mathbf{Y}(\lambda \mathbf{y}.\overline{\mathbf{has}}\,\mathbf{xy}))$
a	np/n	$(\bar{\iota} \to \bar{o}) \to ((\bar{\iota} \to \bar{o}) \to \bar{o})$	$\lambda \mathbf{PQ}.\overline{\exists}(\lambda \mathbf{x}.\mathbf{Px} \mathbin{\overline{\wedge}} \mathbf{Qx})$
child	n	$\bar{\iota} \to \bar{o}$	$\overline{\mathbf{child}}$
he	np	$((\bar{\iota} \to \bar{o}) \to \bar{o})$	$\lambda \mathbf{P}.\mathbf{P}(\text{sel}(\lambda \mathbf{x}.\mathbf{human}\,\mathbf{x} \wedge \mathbf{male}\,x))$
's	$np\backslash(np/n)$	$(\bar{\iota} \to \bar{o}) \to \bar{o} \to (\bar{\iota} \to \bar{o}) \to ((\bar{\iota} \to \bar{o}) \to \bar{o})$	$\lambda \mathbf{YX}.\lambda \mathbf{P}.\mathbf{P}(\widetilde{\text{sel}}\,(\lambda \mathbf{x}.((\mathbf{Xx}) \mathbin{\overline{\wedge}} \mathbf{Y}(\overline{[\![has]\!]}\,\mathbf{x}))))$

Table 2.3: Dynamic interpretations of individual lexical items comprising utterances (7) and (8).

Lexical item	Syntax	Static type	Static interpretation
John	np	$((\iota \to o) \to o)$	$\lambda \text{P.P}\mathbf{j}$, where **j** is a constant
is_back	$np\backslash S$	$((\iota \to o) \to o) \to o$	$\lambda \text{X.X}(\lambda x.\mathbf{back}\,x)$
has	$(np\backslash S)/np$	$((\iota \to o) \to o) \to ((\iota \to o) \to o) \to o$	$\lambda \text{YX.X}(\lambda x.\text{Y}(\lambda y.\mathbf{has}\,xy))$
a	np/n	$(\iota \to o) \to ((\iota \to o) \to o)$	$\lambda \text{PQ}.\exists(\lambda x.\text{P}x \wedge \text{Q}x)$
child	n	$\iota \to o$	**child**
he	np	$((\iota \to o) \to o)$	λP.P?
's	$np\backslash(np/n)$	$(\iota \to o) \to o \to (\iota \to o) \to ((\iota \to o) \to o)$	λYX.λP.P?

Table 2.4: Static interpretations of individual lexical items comprising utterances (7) and (8).

To compute the interpretations of (7) and (8), the individual lexical items are composed according to their syntactic rules to build larger lexical items and then β-reduced to a normal form. The convention of denoting multiple β-reductions from term u to term v by $u \to_\beta^* v$ is adopted here and throughout. We begin with the clauses comprising the utterances.

Example 2.12 (Normalized meanings of *John is back*, *John has a child* and *his child is happy*)**.** [Lebedeva, 2012, p.209, Example 6.20] All reduction steps are shown for the first utterance; for the other two utterances, full details are omitted as they are extensive and judged not to assist the exposition. For *John is back*:

$$\begin{aligned}\overline{[\![\textit{is\_back}]\!]}\,\widetilde{[\![\textit{John}]\!]} &= (\lambda \mathbf{X}.\mathbf{X}(\lambda \mathbf{x}.\overline{\mathbf{back}}\,\mathbf{x}))\lambda \mathbf{P}.\mathbf{P}(\mathrm{sel}(\text{named "John"}))\\ &\to_\beta (\lambda \mathbf{P}.\mathbf{P}(\mathrm{sel}(\text{named "John"})))(\lambda \mathbf{x}.\overline{\mathbf{back}}\,\mathbf{x})\\ &\to_\beta (\lambda \mathbf{x}.\overline{\mathbf{back}}\,\mathbf{x})(\mathrm{sel}(\text{named "John"}))\\ &\to_\beta \overline{\mathbf{back}}\,(\mathrm{sel}(\text{named "John"}))\end{aligned}$$

Substituting in the dynamic term abbreviated by $\overline{\mathbf{back}}$:

$$\begin{aligned}\overline{\mathbf{back}}\,(\mathrm{sel}(\text{named "John"})) = \lambda e\phi.\mathbf{back}\,&\big(\mathrm{sel}(\text{named "John"})e\big)\\ &\wedge \phi\left(\mathrm{upd}(\mathbf{back}\,\big(\mathrm{sel}(\text{named "John"})e\big), e\right)\end{aligned} \tag{2.11}$$

For *John has a child*:

$$\begin{aligned}
&\overline{[\![has]\!]}\,(\overline{[\![a]\!]}\,\overline{[\![child]\!]}\,)\widetilde{[\![John]\!]}\\
&= \lambda \mathbf{YX}.\mathbf{X}(\lambda \mathbf{x}.\mathbf{Y}(\lambda \mathbf{y}.\overline{\mathbf{has}}\,\mathbf{xy}))(\overline{[\![a]\!]}\,\overline{[\![child]\!]}\,)\widetilde{[\![John]\!]}\\
&\rightarrow^*_\beta \widetilde{[\![John]\!]}\,\left(\lambda \mathbf{x}.(\overline{[\![a]\!]}\,\overline{[\![child]\!]}\,)(\lambda \mathbf{y}.\overline{\mathbf{has}}\,\mathbf{xy})\right)\\
&= \left(\lambda \mathbf{P}.\mathbf{P}(\mathrm{sel}(\text{named ``John''}))\right)\left(\lambda \mathbf{x}.(\overline{[\![a]\!]}\,\overline{[\![child]\!]}\,)(\lambda \mathbf{y}.\overline{\mathbf{has}}\,\mathbf{xy})\right)\\
&\rightarrow_\beta \lambda \mathbf{x}.(\overline{[\![a]\!]}\,\overline{[\![child]\!]}\,)(\lambda \mathbf{y}.\overline{\mathbf{has}}\,\mathbf{xy})(\mathrm{sel}(\text{named ``John''}))\\
&\rightarrow_\beta (\overline{[\![a]\!]}\,\overline{[\![child]\!]}\,)\left(\lambda \mathbf{y}.\overline{\mathbf{has}}\,(\mathrm{sel}(\text{named ``John''}))\mathbf{y}\right)\\
&= \left(\lambda \mathbf{PQ}.\overline{\exists}(\lambda \mathbf{x}.\mathbf{Px}\mathbin{\overline{\wedge}}\mathbf{Qx})\overline{\mathbf{child}}\right)\left(\lambda \mathbf{y}.\overline{\mathbf{has}}\,(\mathrm{sel}(\text{named ``John''}))\mathbf{y}\right)\\
&\rightarrow_\beta \left(\lambda \mathbf{Q}.\overline{\exists}\left(\lambda \mathbf{x}.\overline{\mathbf{child}}\,\mathbf{x}\mathbin{\overline{\wedge}}\mathbf{Qx}\right)\right)\left(\lambda \mathbf{y}.\overline{\mathbf{has}}\,(\mathrm{sel}(\text{named ``John''}))\mathbf{y}\right)\\
&\rightarrow_\beta \overline{\exists}\left(\lambda \mathbf{x}.\overline{\mathbf{child}}\,\mathbf{x}\mathbin{\overline{\wedge}}\left(\lambda \mathbf{y}.\overline{\mathbf{has}}\,(\mathrm{sel}(\text{named ``John''}))\mathbf{y}\right)\mathbf{x}\right)\\
&\rightarrow_\beta \overline{\exists}\left(\lambda \mathbf{x}.\overline{\mathbf{child}}\,\mathbf{x}\mathbin{\overline{\wedge}}\overline{\mathbf{has}}\,(\mathrm{sel}(\text{named ``John''}))\mathbf{x}\right)
\end{aligned}$$

Substituting in the dynamic terms and performing β-reductions:

$$\begin{aligned}
&\overline{\exists}\left(\lambda \mathbf{x}.\overline{\mathbf{child}}\,\mathbf{x}\mathbin{\overline{\wedge}}\overline{\mathbf{has}}\,(\mathrm{sel}(\text{named ``John''}))\mathbf{x}\right)\\
&\rightarrow^*_\beta \lambda e\phi.\exists\Bigg(\lambda k.\mathbf{child}\,k \wedge \mathbf{has}\,(\mathrm{sel}(\text{named ``John''})(\mathrm{upd}(\mathbf{child}\,k,e)))k\wedge\\
&\phi\left(\mathrm{upd}\left(\mathbf{has}\,(\mathrm{sel}(\text{named ``John''})(\mathrm{upd}(\mathbf{child}\,k,e)))k,\mathrm{upd}(\mathbf{child}\,k,e)\right)\right)\Bigg)
\end{aligned}\tag{2.12}$$

Finally, for *his child is happy*:

$$\begin{aligned}
&\overline{[\![is_happy]\!]}\,(\widetilde{[\![\text{'}s]\!]}\,\widetilde{[\![he]\!]}\,\overline{[\![child]\!]}\,)\\
&\rightarrow^*_\beta \overline{\mathbf{happy}}\left(\widetilde{\mathrm{sel}}\left(\lambda \mathbf{x}.\left(\overline{\mathbf{child}}\,\mathbf{x}\mathbin{\overline{\wedge}}(\overline{\mathbf{poss}}\,\mathbf{x})(\mathrm{sel}(\lambda x.\mathbf{male}\wedge\mathbf{human}\,x))\right)\right)\right)\\
&\rightarrow^*_\beta \lambda e\phi.\mathbf{happy}\left(\mathrm{sel}\left(\lambda x.\mathbf{child}\,x\wedge\mathbf{has}\,(\mathrm{sel}(\lambda y.\mathbf{male}\,y\wedge\mathbf{human}\,y)e)x\right)e\right)\wedge\\
&\phi\left(\mathrm{upd}\left(\mathbf{happy}\left(\mathrm{sel}\left(\lambda x.\mathbf{child}\,x\wedge\mathbf{has}\,(\mathrm{sel}(\lambda y.\mathbf{male}\,y\wedge\mathbf{human}\,y)e)x\right)e\right),e\right)\right)
\end{aligned}\tag{2.13}$$

With these normal forms (2.11), (2.12) and (2.13) we can compute the interpretations of (7) and (8). Again, the diminishing returns of the examples are accounted for by progressively omitting more details.

Example 2.13 (*If John is back then his child is happy*, sentence interpretation)**.** [Lebedeva, 2012, p.210, Example 6.21]. The meaning $\mathbf{S}_{(7)}$ of sentence (7) is computed by normalizing the following term:

$$\begin{aligned}\mathbf{S}_{(7)} &= \widetilde{[\![\mathit{if}\ldots\mathit{then}\ldots]\!]}\left(\overline{[\![\mathit{is\_back}]\!]}\,\widetilde{[\![\mathit{John}]\!]}\right)\left(\overline{[\![\mathit{is\_happy}]\!]}\,(\widetilde{[\![\mathit{'s}]\!]}\,\widetilde{[\![\mathit{he}]\!]}\,\overline{[\![\mathit{child}]\!]}\,)\right)\\ &= \left(\lambda\mathbf{PQ}.\overline{\neg}\,(\mathbf{P}\mathbin{\overline{\wedge}}\overline{\neg}\mathbf{Q})\right)\left(\overline{[\![\mathit{is\_back}]\!]}\,\widetilde{[\![\mathit{John}]\!]}\right)\left(\overline{[\![\mathit{is\_happy}]\!]}\,(\widetilde{[\![\mathit{'s}]\!]}\,\widetilde{[\![\mathit{he}]\!]}\,\overline{[\![\mathit{child}]\!]}\,)\right)\\ &\to^*_\beta \overline{\neg}\left(\overline{[\![\mathit{is\_back}]\!]}\,\widetilde{[\![\mathit{John}]\!]}\;\overline{\wedge}\,\overline{\neg}\left(\overline{[\![\mathit{is\_happy}]\!]}\,(\widetilde{[\![\mathit{'s}]\!]}\,\widetilde{[\![\mathit{he}]\!]}\,\overline{[\![\mathit{child}]\!]}\,)\right)\right)\end{aligned} \tag{2.14}$$

This proceeds by considering progressively larger subterms of (2.14), starting with the following:

$$\begin{aligned}&\overline{\neg}\left(\overline{[\![\mathit{is\_happy}]\!]}\,(\widetilde{[\![\mathit{'s}]\!]}\,\widetilde{[\![\mathit{he}]\!]}\,\overline{[\![\mathit{child}]\!]}\,)\right)\\ &\to^*_{eval} \lambda e\phi.\neg\mathbf{happy}\,\left(\mathrm{sel}\,(\lambda x.\mathbf{child}\,x \wedge \mathbf{has}\,(\mathrm{sel}(\lambda y.\mathbf{male}\,y \wedge \mathbf{human}\,y)e)x)\;e\right) \wedge \phi e\end{aligned} \tag{2.15}$$

Remark 2.14. Comparing term (2.15) with the non-negated (2.13), in addition to negating the subterm $\mathbf{happy}\,(\ldots)$, dynamic negation also ensures the continuation of the discourse is not interpreted in a context updated with $\mathbf{happy}\,(\ldots)$. This corresponds to the meaning of "If A then B", which does not assert the truth of B.

Continuing the computation:

$$\begin{aligned}&\overline{[\![\mathit{is\_back}]\!]}\,\widetilde{[\![\mathit{John}]\!]}\;\overline{\wedge}\,\overline{\neg}\left(\overline{[\![\mathit{is\_happy}]\!]}\,(\widetilde{[\![\mathit{'s}]\!]}\,\widetilde{[\![\mathit{he}]\!]}\,\overline{[\![\mathit{child}]\!]}\,)\right)\\ &\to^*_\beta \lambda e\phi.\mathbf{back}\,\left(\mathrm{sel}(\text{named "John"})e\right)\\ &\wedge \neg\mathbf{happy}\,\left(\mathrm{sel}\,(\lambda x.\mathbf{child}\,x \wedge \mathbf{has}\,(\mathrm{sel}(\lambda y.\mathbf{male}\,y \wedge \mathbf{human}\,y)e_b)x)\;e_b\right) \wedge \phi e_b\end{aligned} \tag{2.16}$$

where $e_b = \mathrm{upd}\left(\mathbf{back}\,\left(\mathrm{sel}(\text{named "John"})e\right), e\right)$.

Remark 2.15. The definition of dynamic conjunction (2.9f) means that in the subterm (2.16), the consequent of the conditional (*his child is happy*) is interpreted in a context containing the antecedent of the conditional (*John is back*). This corresponds to the meaning of "If A then B", which asserts B only supposing that A is true.

Finally, the interpretation of the entire sentence:

$$\mathbf{S}_{(7)} = \overline{\neg}\left(\overline{[\![is\_back]\!]}\,\widetilde{\overline{[\![John]\!]}}\ \overline{\wedge}\,\overline{\neg}\left(\overline{[\![is\_happy]\!]}\,(\widetilde{[\![\text{'}s]\!]}\,\widetilde{[\![he]\!]}\,\overline{[\![child]\!]}\,)\right)\right)$$
$$\rightarrow^*_\beta \lambda e\phi.\neg\Big(\mathbf{back}\ (\mathrm{sel}(\text{named "John"})e)$$
$$\wedge \neg\mathbf{happy}\ \Big(\mathrm{sel}\ (\lambda x.\mathbf{child}\, x \wedge \mathbf{has}\ (\mathrm{sel}(\lambda y.\mathbf{male}\, y \wedge \mathbf{human}\, y)e_b)x)\ e_b\Big)\ \Big) \wedge \phi e \tag{2.17}$$

Remark 2.16. The outer dynamic negation in the definition of dynamic conditional (2.10b) means in the final interpretation (2.17) of $\mathbf{S}_{(7)}$, the continuation of the discourse does not receive the context updated with the antecedent of the conditional (nor the consequent, as per Remark 2.14 This corresponds to the meaning of "If A then B" as not requiring either A or B to hold.

Example 2.17 (*If John has a child then his child is happy*, sentence interpretation)**.** [Lebedeva, 2012, Example 6.25, p.219]. The meaning $\mathbf{S}_{(8)}$ of sentence ((8)) is computed by normalizing the term (2.18), proceeding as before and resulting in the final interpretation 2.19.

$$\mathbf{S}_{(8)} \rightarrow^*_\beta \overline{\neg}\left(\overline{[\![has]\!]}\,(\overline{[\![a]\!]}\,\overline{[\![child]\!]}\,)\widetilde{\overline{[\![John]\!]}}\ \overline{\wedge}\,\overline{\neg}\left(\overline{[\![is\_happy]\!]}\,(\widetilde{[\![\text{'}s]\!]}\,\widetilde{[\![he]\!]}\,\overline{[\![child]\!]}\,)\right)\right) \tag{2.18}$$

For clarity, let $e_k = \mathrm{upd}(\mathbf{child}\, y, e)$ in term (2.12), then:

$$\overline{[\![has]\!]}\,(\overline{[\![a]\!]}\,\overline{[\![child]\!]}\,)\widetilde{\overline{[\![John]\!]}}\ \overline{\wedge}\,\overline{\neg}\left(\overline{[\![is\_happy]\!]}\,(\widetilde{[\![\text{'}s]\!]}\,\widetilde{[\![he]\!]}\,\overline{[\![child]\!]}\,)\right)$$
$$\rightarrow^*_\beta \lambda e\phi.\exists\Big(\lambda k.\mathbf{child}\, k \wedge \mathbf{has}\ (\mathrm{sel}(\text{named "John"})e_k)k$$
$$\wedge \neg\mathbf{happy}\ \Big(\mathrm{sel}\ (\lambda x.\mathbf{child}\, x \wedge \mathbf{has}\ (\mathrm{sel}(\lambda y.\mathbf{male}\, y \wedge \mathbf{human}\, y)e_{kh})x)\ e_{kh}\Big)\ \Big) \wedge \phi e_{kh}$$

where $e_{kh} = \mathrm{upd}\left(\mathbf{has}\ (\mathrm{sel}(\text{named "John"})e_k)\, k, e_k\right)$. As per Remark 2.15 for $\mathbf{S}_{(7)}$, the consequent *his child is happy* is interpreted in a context containing the antecedent *John* *has a child*.

$$\begin{aligned} &\mathbf{S}_{(8)} \overline{\neg} \left(\overline{[\![has]\!]} \left(\overline{[\![a]\!]}\, \overline{[\![child]\!]} \right) \widetilde{[\![John]\!]} \;\overline{\wedge}\; \overline{\neg} \left(\overline{[\![is\_happy]\!]} \left(\widetilde{[\![\text{'}s]\!]}\, \widetilde{[\![he]\!]}\, \overline{[\![child]\!]} \right) \right) \right) \\ &\quad \rightarrow_{\beta}^{*} \lambda e\phi.\neg\exists \Bigg(\lambda k.\mathbf{child}\, k \wedge \mathbf{has}\, (\mathrm{sel}(\text{named “John”})e_k)k \\ &\quad \wedge \neg\mathbf{happy}\, \Big(\mathrm{sel}\, \big(\lambda x.\mathbf{child}\, x \wedge \mathbf{has}\, (\mathrm{sel}(\lambda y.\mathbf{male}\, y \wedge \mathbf{human}\, y)e_{kh})x\big)\, e_{kh}\Big) \Bigg) \wedge \phi e \end{aligned} \tag{2.19}$$

As per Remark 2.16 for $\mathbf{S}_{(7)}$, the outer negation means the continuation does not receive an updated context.

2.3.3 Revising the Definition of Dynamic Negation

Before continuing, we return to Remark 2.14 and demonstrates that this is not the desired interpretation of dynamic negation, revising the definition accordingly. Consider a third utterance (9):

(9) John is back and his child is happy.

It is interpretated as term (2.20):

$$\begin{aligned} &\mathbf{S}_{(9)} = \widetilde{[\![and]\!]} \left(\overline{[\![is\_back]\!]}\, \widetilde{[\![John]\!]} \right) \left(\overline{[\![is\_happy]\!]} \left(\widetilde{[\![\text{'}s]\!]}\, \widetilde{[\![he]\!]}\, \overline{[\![child]\!]} \right) \right) \\ &\quad \rightarrow_{\beta}^{*} \lambda e\phi.\mathbf{back}\, \big(\mathrm{sel}(\text{named “John”})e\big) \\ &\quad \wedge \mathbf{happy}\, \Big(\mathrm{sel}\, \big(\lambda x.\mathbf{child}\, x \wedge \mathbf{has}\, (\mathrm{sel}(\lambda y.\mathbf{male}\, y \wedge \mathbf{human}\, y)e_b)x\big)\, e_b\Big) \wedge \phi e_{bh} \end{aligned} \tag{2.20}$$

with abbreviated contexts:

$$\begin{aligned} e_b &= \mathrm{upd}\Big(\mathbf{back}\, \big(\mathrm{sel}(\text{named “John”})e\big), e\Big) \\ e_{bh} &= \mathrm{upd}\Bigg(\mathbf{happy}\, \Big(\mathrm{sel}\, \big(\lambda x.\mathbf{child}\, x \wedge \mathbf{has}\, (\mathrm{sel}(\lambda y.\mathbf{male}\, y \wedge \mathbf{human}\, y)e_b)x\big)\, e_b\Big), e_b\Bigg) \end{aligned}$$

The context passed to the rest of the discourse, e_{bh}, contains not just the presupposition of the sentence, but the content of the sentence itself. In contrast, the interpretations of the conditional utterances (2.17) and (2.19) pass the unupdated context parameter e to the continuation. While we wish not to pass referents introduced by *if* to the rest of the discourse, we do wish to include the content of the conditional. The epistemic status of a sentence containing a connective with a logical interpretation involving negation – such as *if…then* – is the same as one with a logical interpretation not involving negation – such as *and*.

To see how this has an impact on interpreting discourse, consider the following conversation:

(10) A: If John is back then his child is happy.
B: Is John back?
C: His child is happy.

C's statement communicates more than what is explicitly said: it is reasonable for A and B to suppose C is providing information relevant to whether John is back, not making an unrelated assertion, and so they may conclude that C believes John is back. The current definition of negation is a barrier to making this interpretation since it admits A's utterance to the discourse content but not the context in which proceeding utterances are interpreted.

To address this, we replace the original definition of dynamic negation,

$$\lambda e\phi.\neg \mathbf{A}e(\lambda e.\top) \wedge \phi e$$

with the following definition.

Definition 2.18 (Revised definition of dynamic negation)**.** Let **A** be a dynamic proposition, type $(\gamma \to (\gamma \to o) \to o)$, then dynamic negation is defined as follows:

$$\bar{\neg} := \lambda e\phi.\neg \mathbf{A}e(\lambda e.\top) \wedge \phi \left(\mathrm{upd}(\neg \mathbf{A}e(\lambda e.\top), e)\right)$$

The interpretations from the preceding examples are corrected accordingly. The interpretation of (7) by term (2.17) is corrected by altering the last subterm:

$$\lambda e\phi.\neg \Bigg(\mathbf{back}\ \left(\mathrm{sel}(\mathrm{named}\ \text{“John”})e\right)$$
$$\wedge \neg \mathbf{happy}\ \left(\mathrm{sel}\ \left(\lambda x.\mathbf{child}\, x \wedge \mathbf{has}\ (\mathrm{sel}(\lambda y.\mathbf{male}\, y \wedge \mathbf{human}\, y)e_b)x\right)\ e_b\right) \Bigg) \wedge \phi e_i \quad (2.21)$$
$$\text{where } e_i = \mathrm{upd}\Bigg(\neg \Big(\mathbf{back}\ \left(\mathrm{sel}(\mathrm{named}\ \text{“John”})e\right)$$
$$\wedge \neg \mathbf{happy}\ \left(\mathrm{sel}\ \left(\lambda x.\mathbf{child}\, x \wedge \mathbf{has}\ (\mathrm{sel}(\lambda y.\mathbf{male}\, y \wedge \mathbf{human}\, y)e_b)x\right)\ e_b\right) \Big), e \Bigg)$$

The interpretation of (8) by term (2.19) is corrected by altering the last subterm:

$$\lambda e\phi.\neg\exists\Big(\lambda k.\textbf{child}\,k \wedge \textbf{has}\,(\text{sel}(\text{named “John”})e_k)k$$
$$\wedge\ \neg\textbf{happy}\ \Big(\text{sel}\,\big(\lambda x.\textbf{child}\,x \wedge \textbf{has}\,(\text{sel}(\lambda y.\textbf{male}\,y \wedge \textbf{human}\,y)e_{kh})x\big)\ e_{kh}\Big)\ \Big) \wedge \phi e_i \tag{2.22}$$

$$\text{where } e_i = \text{upd}\Bigg(\neg\exists\Big(\lambda k.\textbf{child}\,k \wedge \textbf{has}\,(\text{sel}(\text{named “John”})e_k)k$$
$$\wedge\ \neg\textbf{happy}\ \Big(\text{sel}\,\big(\lambda x.\textbf{child}\,x \wedge \textbf{has}\,(\text{sel}(\lambda y.\textbf{male}\,y \wedge \textbf{human}\,y)e_{kh})x\big)\ e_{kh}\Big)\ \Big), e\Bigg)$$

In this interpretation, we see how negation still has the desired effect of blocking access to the individual k outside the scope of the negation, as introduced by *if*, since the context e_k that contains it is not passed to the continuation of the discourse.

2.3.4 Framework $GL\chi$

Lebedeva [2012] adds an exception raising and handling mechanism to GL to account for presuppositions. The resulting framework, $GL\chi$, is defined by adding the following terms to the λ-calculus.

Definition 2.19 ($GL\chi$ terms)**.** The set of λ-terms by which GL is extended to define the terms of $GL\chi$ is given by the following formal grammar:

$$t := x \mid k \mid (Et) \mid \lambda x.t \mid (tt) \mid (\text{raise } t) \mid t \texttt{ handle } (Ex) \texttt{ with } t$$

where x is a variable, k is a constant and E is an exception constructor.

A new type is added to the calculus: the type χ of exceptions.

Definition 2.20 ($GL\chi$ types)**.** The set T of types of $GL\chi$ is given by the following formal grammar:

$$T = \iota \mid o \mid \gamma \mid \chi \mid T \to T$$

The typing judgements for $GL\chi$ are those given for GL in Definition (2.8), along with the following rules.

Definition 2.21 ($GL\chi$ typing judgements)**.** Let α and β be arbitrary types and E be an exception constructor of type $\beta \to \chi$. The judgement $t : \tau$ is derivable in $GL\chi$ from the basis Δ if $\Delta \vdash t : \tau$ can be produced using the rules in (2.8) and the following rules:

$$\frac{}{\Gamma \vdash E : \beta \to \chi}$$

$$\frac{\Gamma \vdash e : \chi}{\Gamma \vdash \text{raise } e : \alpha}$$

$$\frac{\Gamma \vdash t_1 : \alpha \qquad \Gamma \vdash E : \beta \to \chi \qquad \Gamma, x : \beta \vdash t_2 : \alpha}{\Gamma \vdash t_1 \texttt{ handle } Ex \texttt{ with } t_2 : \alpha}$$

Lebedeva proceeds by defining strong and weak evaluation rules for the new calculus [Lebedeva, 2012, Definitions 6.11, 6.12, 6.13]. We omit the details, but introduce the notation $u \to_{eval} v$ to denote that u evaluates to v by these rules.

With the addition of exception raising and handling to the formalism, we may now define the special constant sel.

Definition 2.22 ($GL\chi$ selection function)**.** Let P be a term of type $(\iota \to o)$ and e be a term of type o. Then sel P e is defined as follows:

$$\text{sel P } e := \begin{cases} \text{choose } \{\texttt{a} \mid e \vdash \text{P}\texttt{a}\} & \text{if } \{e \vdash \text{P}\texttt{a}\} \neq \emptyset \\ \text{raise } (\texttt{AbsentIndividualExc P}) & \text{otherwise} \end{cases}$$

This function returns an individual with the property P if one can be found in the context, and selected according to the oracle function choose if many are found. If there is no such individual, then the exception `AbsentIndividualExc` carrying the property P is raised. Handling of this exception occurs with discourse update, defined in the next section.

Note that this definition, unlike those prior, depends on a particular context structure, namely a conjunction of propositions. Although $GL\chi$ is largely a generic framework by the way context is incorporated, parts of it depend on a specific structure of context – to actually use a framework on sentences means making it messy. Identifying which terms of the generic framework have been given special interpretations based on the specific context structure is necessary before using a new one. With this structure fixed, the other special constant upd is given the following definition.

Definition 2.23 ($GL\chi$ context update)**.** Let P be a term of type $(\iota \to o)$ and e be a term of type o, then context update is defined as follows:

$$\text{upd}(\text{P}e, e) := \text{P}e \wedge e$$

2.3.4.1 Discourse Update

So far the framework has been built without reference to how discourse is interpreted. Just as the meaning of a sentence is computed compositionally, so too is the meaning of a discourse. de Groote [2006] gets the meaning of a discourse by composing the meaning of sentences according to the following equation, where S is a sentence of type $(\gamma \to (\gamma \to o) \to o)$ and D is a discourse of type $(\gamma \to (\gamma \to o) \to o)$:

$$D.S \coloneqq \lambda e\phi.De(\lambda e'.Se'\phi) \tag{2.23}$$

This is essentially dynamic conjunction of sentences.

Lebedeva [2012] alters this by conceiving of the term *discourse* as denoting a sequence of sentences in a *concrete* context. This pragmatic view means discourse **D** has type $((\gamma \to o) \to o)$, parametrized only by the future of the discourse. Lebedeva's preliminary definition of discourse update is given by the following function, (2.23) without the context parameter:

$$\mathrm{dupd}\ \mathbf{D}\ \mathbf{S} \coloneqq \lambda\phi.\mathbf{D}e(\lambda e'.\mathbf{S}e'\phi)$$

This definition is preliminary because discourse update is also the location of exception handling: the interaction between the meaning of a sentence containing presupposed meaning and the interpretation of the discourse motivates placing it here. This is achieved by the following definition.

Definition 2.24 ($GL\chi$ discourse update)**.** [Lebedeva, 2012, Definition 6.6, p.183]

$$\mathrm{dupd}\ \mathbf{D}\ \mathbf{S} \coloneqq \lambda\phi.\mathbf{D}(\lambda e.\mathrm{gacc}\ \mathbf{S}\ e\ \phi) \tag{2.24a}$$

$$\mathrm{gacc}\ \mathbf{S}\ e\ \phi \coloneqq \mathbf{S}\ e\ \phi \tag{2.24b}$$

$$\texttt{handle}\ (\texttt{AbsentIndividualExc}\ \mathrm{P})\ \texttt{with}$$

$$\exists(\lambda x.(\mathrm{P}x) \wedge \mathrm{gacc}\ \mathbf{S}\ (\mathrm{upd}(\mathrm{P}x, e))\ \phi) \tag{2.24c}$$

The function gacc, for *global accommodation*, returns $\mathbf{S}\ e\ \phi$ if there are no exceptions to handle. If there is an exception `AbsentIndividualExc` P, it is handled by the creation of a new individual x with the property P. The handler also includes a recursive call of gacc with an updated context $\mathrm{upd}(\mathrm{P}x, e)$ including the new individual. With this recursive call, the desired individual can be found by the selection function and the previous exception is no longer raised.

Processing a discourse requires an initial discourse before any sentences have been added. This is given by the following term:

$$\mathbf{D}_0 := \lambda\phi.\phi c$$

for a context c. For the interpretation of a sentence $\mathbf{S}$ in the empty discourse $\mathbf{D}_0$, the following result will be useful:

$$\text{dupd } \mathbf{D}_0 \ \mathbf{S} \rightarrow^*_\beta \lambda\phi.\text{gacc } \mathbf{S} \ c \ \phi \tag{2.25}$$

This is true by the following reductions:

$$\begin{aligned}
\text{dupd } \mathbf{D}_0 \ \mathbf{S} &= \lambda\phi.\mathbf{D}_0(\lambda e.\text{gacc } \mathbf{S} \ e \ \phi) \\
&= \lambda\phi.(\lambda\phi.\phi \mathrm{C})(\lambda e.\text{gacc } \mathbf{S} \ e \ \phi) \\
&\rightarrow_\beta \lambda\phi.(\lambda e.\text{gacc } \mathbf{S} \ e \ \phi)c \\
&\rightarrow_\beta \lambda\phi.\text{gacc } \mathbf{S} \ c \ \phi
\end{aligned}$$

2.3.4.2 Examples: Presupposition Projection

To become familiar with how exception handling and discourse update work in $GL\chi$, and to see how the formalism accounts for presupposition related phenomena, we return to utterances (7) and (8) to consider their interpretations in a discourse. Presupposition is meaning assumed by an utterance for it to be meaningful. Presuppositions are associated with particular lexical items, called *presupposition triggers*. When the hearer encounters presupposed content that is not in their context, *presupposition accommodation* occurs. This refers to the content being added to their context, in addition to the regular content of the utterance.

Presuppositions can occur in complex utterances: sentences with multiple propositions joined by discourse connectives. When a presupposition of a component of a sentence is inherited by the entire sentence, it is an instance of *presupposition projection*. Alternatively, this embedding may mean the presupposition no longer emerges and is said to have been *cancelled*.

Utterances (7), (8) and (9) share the proposition "his child is happy", containing the lexical item 'his', which is both anaphoric – referring back to John – and triggers the presupposition that John has a child. In (7) and (9), this presupposition is projected to the entire sentence, while in (8) it is cancelled by the antecedent. The presupposition projection behaviour does not emerge on the level of sentence, because the presupposed content is accommodated or not is contingent on whether it is contained in the context. To show how $GL\chi$ accounts for presupposition emergence and non-emergence in these utterances, we compare their discourse-level interpretation.

The examples from *Presupposition Projection in Conditionals* [Lebedeva, 2012, Section 6.4] are reviewed here to highlight how the definition of dynamic conditional in terms of dynamic negation and dynamic conjunction captures presupposition projection. A discussion of these examples under the changes to the framework proposed in *Directions for Further Development of the Framework: Conversational Implicatures* [Lebedeva, 2012, Section 7.2] is foreshadowed by also remarking on the behaviour of upd and dupd. They differ from the original presentation by using sentence interpretations with the revised dynamic negation definition. This illustrates how the desired presupposition projection behaviour is still achieved with the new definition. These examples are presented with respect to the following discourse **D** and context C:

$$\begin{aligned}
\mathrm{C} &= \forall x.(\text{named "John"}x) \vee (\text{named "Mary"}x) \rightarrow \textbf{human}\, x \\
&\quad \wedge \forall x.(\text{named "John"}x) \rightarrow \textbf{male}\, x \\
&\quad \wedge \forall x.(\text{named "Mary"}x) \rightarrow \textbf{female}\, x \\
\mathbf{D} &= \lambda\phi.\exists \left(\lambda j.(\text{named "John"}j \wedge \phi\, (\mathrm{upd}(\text{named "John"}j, \mathrm{C}))\right) \\
&= \lambda\phi.\exists \left(\lambda j.(\text{named "John"}j \wedge \phi \mathrm{C}_j\right)
\end{aligned}$$

where $\mathrm{C}_j = \big(\mathrm{upd}(\text{named "John"}j, \mathrm{C})\big)$.

Example 2.25 (*If John is back, then his child is happy*, discourse interpretation)**.** [Lebedeva, 2012, Example 6.22, p.211] Suppose (7) is uttered at the start of a discourse, conducted with the common knowledge contained in the context C. This interpreted as **D** updated with $\mathbf{S}_{(7)}$:

$$\begin{aligned}
\mathrm{dupd}\ \mathbf{D}\ \mathbf{S}_{(7)} &= \lambda\phi.\mathbf{D}(\lambda e.\mathrm{gacc}\ \mathbf{S}_{(7)}\ e\ \phi) \\
&= \lambda\phi.\Big(\lambda\phi.\exists\Big(\lambda j.\text{named “John”}j \wedge \phi \mathrm{C}_j\Big)\Big)(\lambda e.\mathrm{gacc}\ \mathbf{S}_{(7)}\ e\ \phi) \\
&\to_\beta \lambda\phi.\exists\Big(\lambda j.\text{named “John”}j \wedge (\lambda e.\mathrm{gacc}\ \mathbf{S}_{(7)}\ e\ \phi)\mathrm{C}_j\Big) \\
&\to_\beta \lambda\phi.\exists\Big(\lambda j.\text{named “John”}j \wedge \mathrm{gacc}\ \mathbf{S}_{(7)}\ \mathrm{C}_j\ \phi\Big)
\end{aligned} \tag{2.26}$$

The computation continues in the subterm $\mathbf{S}_{(7)}\ \mathrm{C}_j\ \phi$:

$$\begin{aligned}
\mathbf{S}_{(7)}\ \mathrm{C}_j\ \phi = &\Big(\lambda e\phi.\neg\Big(\mathbf{back}\ \big(\mathrm{sel}(\text{named “John”})e\big) \wedge \neg\mathbf{happy}\ \Big(\mathrm{sel}\ \big(\lambda x.\mathbf{child}\ x \wedge \mathbf{has}\ (\mathrm{sel}(\lambda y.\mathbf{male}\ y \wedge \mathbf{human}\ y)e_b)x\big)\ e_b\Big)\Big) \wedge \phi e_i\Big)\mathrm{C}_j\phi \\
\to^*_\beta &\neg\Big(\mathbf{back}\ \big(\mathrm{sel}(\text{named “John”})\mathrm{C}_j\big) \wedge \neg\mathbf{happy}\ \Big(\mathrm{sel}\ \big(\lambda x.\mathbf{child}\ x \wedge \mathbf{has}\ (\mathrm{sel}(\lambda y.\mathbf{male}\ y \wedge \mathbf{human}\ y)\mathrm{C}_{bj})x\big)\ \mathrm{C}_{bj}\Big)\Big) \wedge \phi \mathrm{C}_{ij}
\end{aligned} \tag{2.27}$$

where $\mathrm{C}_{bj} = \mathrm{upd}\Big(\mathbf{back}\ \big(\mathrm{sel}(\text{named “John”})\mathrm{C}_j\big), \mathrm{C}_j\Big)$

$$\mathrm{C}_{ij} = \mathrm{upd}\Big(\neg\Big(\mathbf{back}\ \big(\mathrm{sel}(\text{named “John”})\mathrm{C}_j\big) \wedge \neg\mathbf{happy}\ \Big(\mathrm{sel}\ \big(\lambda x.\mathbf{child}\ x \wedge \mathbf{has}\ (\mathrm{sel}(\lambda y.\mathbf{male}\ y \wedge \mathbf{human}\ y)\mathrm{C}_{bj})x\big)\ \mathrm{C}_{bj}\Big)\Big), \mathrm{C}_j\Big)$$

Since $\mathrm{C}_j \vdash$ named “John”j, the selection function call $\mathrm{sel}(\text{named “John”})\mathrm{C}_j$ returns j. Similarly, $\mathrm{sel}(\lambda y.\mathbf{male}\ y \wedge \mathbf{human}\ y)\mathrm{C}_{bj}$

evaluates to j since $\mathrm{C}_{bj} \vdash \mathbf{male}\, j \wedge \mathbf{human}\, j$. Performing these substitions in 2.27:

$$\neg\left(\mathbf{back}\, j \wedge \neg\mathbf{happy}\ \left(\mathrm{sel}\ (\lambda x.\mathbf{child}\ x \wedge \mathbf{has}\ j\ x)\ \mathrm{C}_{bj}\right)\right) \wedge \phi\mathrm{C}_{ij} \tag{2.28}$$

$$\text{where } \mathrm{C}_{bj} = \mathrm{upd}\left(\mathbf{back}\, j, \mathrm{C}_j\right)$$

$$\mathrm{C}_{ij} = \mathrm{upd}\left(\neg\left(\mathbf{back}\, j \wedge \neg\mathbf{happy}\ \left(\mathrm{sel}\ (\lambda x.\mathbf{child}\ x \wedge \mathbf{has}\ j\ x)\ \mathrm{C}_{bj}\right)\right), \mathrm{C}_j\right)$$

In the case of $\mathrm{sel}\ (\lambda x.\mathbf{child}\ x \wedge \mathbf{has}\ j\ x)\ \mathrm{C}_{bj}$, there is no $\mathtt{a}$ such that $\mathrm{C}_{bj} \vdash \mathbf{child}\ \mathtt{a} \wedge \mathbf{has}\ j\ \mathtt{a}$, so an exception is raised and propagated:

$$\neg\left(\mathbf{back}\, j \wedge \neg\mathbf{happy}\ (\mathrm{raise}\ \mathtt{AbsentIndividualExc}\ (\lambda x.\mathbf{child}\ x \wedge \mathbf{has}\ j\ x))\right) \wedge \phi\mathrm{C}_{ij}$$

$$\rightarrow_{eval} \mathrm{raise}\ \mathtt{AbsentIndividualExc}\ (\lambda x.\mathbf{child}\ x \wedge \mathbf{has}\ j\ x) \tag{2.29}$$

Substituting (2.29) back into the main term (2.26):

$$\lambda\phi.\exists\left(\lambda j.\text{named "John"} j \wedge \mathrm{gacc}\ \mathbf{S}_{(7)}\ \mathrm{C}_j\ \phi\right)$$

$$\rightarrow_{eval} \lambda\phi.\exists\left(\lambda j.\text{named "John"} j \wedge \mathrm{raise}\ \mathtt{AbsentIndividualExc}\ (\lambda x.\mathbf{child}\ x \wedge \mathbf{has}\ j\ x)\ \mathtt{handle}\ \ldots\right)$$

$$\rightarrow_{eval} \lambda\phi.\exists\left(\lambda j.\text{named "John"} j \wedge \exists\left(\lambda k.\ (\lambda x.\mathbf{child}\ x \wedge \mathbf{has}\ j\ x)\ k \wedge \mathrm{gacc}\ \mathbf{S}_{(7)}\ \left(\mathrm{upd}\left((\lambda x.\mathbf{child}\ x \wedge \mathbf{has}\ j\ x)\ k, \mathrm{C}_j\right)\right)\ \phi\right)\right) \quad \text{(by 3.1c)}$$

(2.30)

Remark 2.26. The exception handler for an absent individual exception corresponds to presupposition accommodation – in the case of *his child is happy*, that John has a child. If an individual satisfying the property of being John's child, there would still be a presupposition but it would not need to be accommodated – the presupposition is encoded in the lexical item but accommodation is by an exception handler in the discourse.

Let $C_{kj} = \text{upd}\left(\textbf{child}\, k \wedge \textbf{has}\, j\, k, C_j\right)$. The computation continues in the following subterm, where because of the updated context C_{kj}, an exception is no longer raised:

$$\begin{aligned}
&\text{gacc}\ \mathbf{S}_{(7)}\ C_{kj}\ \phi \qquad (2.31)\\
&\to^*_{eval} \mathbf{S}_{(7)}\ \left(\text{upd}\left(\textbf{child}\, k \wedge \textbf{has}\, j\, k, C_j\right)\right)\ \phi\ \texttt{handle}\ \ldots\\
&= \left(\lambda e\phi.\neg\left(\textbf{back}\ (\text{sel}(\text{named ``John''})e) \wedge \neg\textbf{happy}\ \left(\text{sel}\ (\lambda x.\textbf{child}\, x \wedge \textbf{has}\ (\text{sel}(\lambda y.\textbf{male}\, y \wedge \textbf{human}\, y)e_b)x)\ e_b\right)\right) \wedge \phi e_i\right) C_{kj}\phi\\
&\to^*_{\beta} \neg\left(\textbf{back}\ \left(\text{sel}(\text{named ``John''})C_{kj}\right) \wedge \neg\textbf{happy}\ \left(\text{sel}\ \left(\lambda x.\textbf{child}\, x \wedge \textbf{has}\ (\text{sel}(\lambda y.\textbf{male}\, y \wedge \textbf{human}\, y)C_{bkj})x\right)\ C_{bkj}\right)\right) \wedge \phi C_{ikj}\\
&\to^*_{\beta} \neg\textbf{back}\, j \wedge \neg\textbf{happy}\, k \wedge \phi C_{ikj}
\end{aligned}$$

where

$$\begin{aligned}
C_{bkj} &= \text{upd}\left(\textbf{back}\ \left(\text{sel}(\text{named ``John''})C_{kj}\right), C_{kj}\right)\\
&= \text{upd}\left(\textbf{back}\, j, C_{kj}\right)\\
C_{ikj} &= \text{upd}\left(\neg\left(\textbf{back}\ \left(\text{sel}(\text{named ``John''})C_{kj}\right) \wedge \neg\textbf{happy}\ \left(\text{sel}\ \left(\lambda x.\textbf{child}\, x \wedge \textbf{has}\ (\text{sel}(\lambda y.\textbf{male}\, y \wedge \textbf{human}\, y)C_{bkj})x\right)\ C_{bkj}\right)\right), C_{kj}\right)\\
&= \text{upd}\left(\neg\left(\textbf{back}\, j \wedge \neg\textbf{happy}\, k\right), C_{kj}\right)
\end{aligned}$$

So we have:

$$\mathrm{gacc}\ \mathbf{S}_{(7)}\ \mathrm{upd}\left(\mathbf{child}\,k \wedge \mathbf{has}\,j\ k, \mathrm{C}_j\right)\ \phi \rightarrow_{eval} \neg\left(\mathbf{back}\,j \wedge \neg\mathbf{happy}\,k\right) \wedge \phi\left(\mathrm{upd}\left(\neg\left(\mathbf{back}\,j \wedge \neg\mathbf{happy}\,k\right), \mathrm{upd}\left(\mathbf{child}\,k \wedge \mathbf{has}\,j\ k, \mathrm{C}_j\right)\right)\right) \tag{2.32}$$

Substituting the subterm (2.32) back into the main term (2.30) gives the interpretation (2.33) of $\mathbf{S}_{(7)}$ in the discourse $\mathbf{D}$:

$$\begin{aligned}
&\lambda\phi.\exists\left(\lambda j.\text{named “John”}j \wedge \mathrm{gacc}\ \mathbf{S}_{(7)}\ \mathrm{C}_j\ \phi\right)\\
&= \lambda\phi.\exists\left(\lambda j.\text{named “John”}j \wedge \exists\left(\lambda k.\mathbf{child}\,k \wedge \mathbf{has}\,j\ k \wedge \mathrm{gacc}\ \mathbf{S}_{(7)}\ \left(\mathrm{upd}\left(\mathbf{child}\,k \wedge \mathbf{has}\,j\ k, \mathrm{C}_j\right)\right)\ \phi\right)\right)\\
&= \lambda\phi.\exists\Big(\lambda j.\text{named “John”}j \wedge \exists\Big(\lambda k.\mathbf{child}\,k \wedge \mathbf{has}\,j\ k \wedge \neg\left(\mathbf{back}\,j \wedge \neg\mathbf{happy}\,k\right)\\
&\qquad \wedge \phi\left(\mathrm{upd}\left(\neg\left(\mathbf{back}\,j \wedge \neg\mathbf{happy}\,k\right), \mathrm{upd}\left(\mathbf{child}\,k \wedge \mathbf{has}\,j\ k, \mathrm{C}_j\right)\right)\right)\Big)\Big)
\end{aligned} \tag{2.33}$$

Compare this to the interpretation in Lebedeva, which is:

$$\lambda\phi.\exists\left(\lambda j.\text{named “John”}j \wedge \exists\left(\lambda k.\mathbf{child}\,k \wedge \mathbf{has}\,j\ k \wedge \neg\left(\mathbf{back}\,j \wedge \neg\mathbf{happy}\,k\right) \wedge \phi\left(\mathrm{upd}\left(\mathbf{child}\,k \wedge \mathbf{has}\,j\ k, \mathrm{C}_j\right)\right)\right)\right)$$

where the continuation of the discourse receives the context updated with the presupposition of the sentence, but not the content of the sentence.

Example 2.27 (*If John has a child, then his child is happy*, discourse interpretation)**.** [Lebedeva, 2012, Example 6.26, p.212] Suppose sentence (8) is uttered at the start of a discourse, conducted with the common knowledge contained in the context C. This is interpreted as $\mathbf{D}$ updated with $\mathbf{S}_{(8)}$:

$$\begin{aligned}
\mathrm{dupd}\ \mathbf{D}\ \mathbf{S}_{(8)} &= \lambda\phi.\mathbf{D}(\lambda e.\mathrm{gacc}\ \mathbf{S}_{(8)}\ e\ \phi) \\
&= \lambda\phi.\Big(\lambda\phi.\exists\Big(\lambda j.\text{named "John"} j \wedge \phi \mathrm{C}_j\Big)\Big)(\lambda e.\mathrm{gacc}\ \mathbf{S}_{(8)}\ e\ \phi) \\
&\rightarrow_\beta \lambda\phi.\exists\Big(\lambda j.\text{named "John"} j \wedge (\lambda e.\mathrm{gacc}\ \mathbf{S}_{(8)}\ e\ \phi)\mathrm{C}_j\Big) \\
&\rightarrow_\beta \lambda\phi.\exists\Big(\lambda j.\text{named "John"} j \wedge \mathrm{gacc}\ \mathbf{S}_{(8)}\ \mathrm{C}_j\ \phi\Big)
\end{aligned} \tag{2.34}$$

The computation continues in the subterm $\mathbf{S}_{(8)}\ \mathrm{C}_j\ \phi$, recalling that e_k, e_{kh} and e_i were defined in the sentence interpretation:

$$\begin{aligned}
&\mathbf{S}_{(8)}\ \mathrm{C}_j\ \phi \\
&= \Big(\lambda e\phi.\neg\exists\ \Big(\lambda k.\mathbf{child}\, k \wedge \mathbf{has}\ (\mathrm{sel}(\text{named "John"})e_k)k \wedge \neg\mathbf{happy}\ \Big(\mathrm{sel}\ \big(\lambda x.\mathbf{child}\, x \wedge \mathbf{has}\ (\mathrm{sel}(\lambda y.\mathbf{male}\, y \wedge \mathbf{human}\, y)e_{hk})x\big)\ e_{hk}\Big)\Big) \wedge \phi e_i\Big)\mathrm{C}_j\phi \\
&\rightarrow_\beta^* \neg\exists\ \Big(\lambda k.\mathbf{child}\, k \wedge \mathbf{has}\ (\mathrm{sel}(\text{named "John"})\mathrm{C}_{kj})k \wedge \neg\mathbf{happy}\ \Big(\mathrm{sel}\ \Big(\lambda x.\mathbf{child}\, x \wedge \mathbf{has}\ (\mathrm{sel}(\lambda y.\mathbf{male}\, y \wedge \mathbf{human}\, y)\mathrm{C}_{hkj})x\Big)\ \mathrm{C}_{hkj}\Big)\Big) \wedge \phi\mathrm{C}_{ij}
\end{aligned} \tag{2.35}$$

where

$$\mathrm{C}_{kj} = \mathrm{upd}(\mathbf{child}\,k, \mathrm{C}_j)$$
$$\mathrm{C}_{hkj} = \mathrm{upd}\left(\mathbf{has}\ \left(\mathrm{sel}(\mathrm{named\ ``John"})\mathrm{C}_{kj}\right) y, \mathrm{C}_{kj}\right)$$
$$\mathrm{C}_{ij} = \mathrm{upd}\left(\neg\exists\left(\lambda k.\mathbf{child}\,k \wedge \mathbf{has}\,(\mathrm{sel}(\mathrm{named\ ``John"})\mathrm{C}_{kj})k \wedge \neg\mathbf{happy}\ \left(\mathrm{sel}\ \left(\lambda x.\mathbf{child}\ x \wedge \mathbf{has}\ (\mathrm{sel}(\lambda y.\mathbf{male}\,y \wedge \mathbf{human}\,y)\mathrm{C}_{hkj})x\right)\ \mathrm{C}_{hkj}\right)\ \right), \mathrm{C}_j\right)$$

As in Example 2.25, $\mathrm{sel}(\mathrm{named\ ``John"})\mathrm{C}_{kj}$ evaluates to j since $\mathrm{C}_{kj} \vdash$ named "John"j, and $\mathrm{sel}(\lambda y.\mathbf{male}\,y \wedge \mathbf{human}\,y)\mathrm{C}_{hkj}$ evaluates to j since $\mathrm{C}_{hkj} \vdash \mathbf{male}\,j \wedge \mathbf{human}\,j$. Performing these substitutions in subterm (2.35):

$$\neg\exists\left(\lambda k.\mathbf{child}\,k \wedge \mathbf{has}\,j\ k \wedge \neg\mathbf{happy}\ \left(\mathrm{sel}\ \left(\lambda x.\mathbf{child}\ x \wedge \mathbf{has}\ j\ x\right)\ \mathrm{C}_{hkj}\right)\right) \wedge \phi\mathrm{C}_{ij} \tag{2.36}$$

where $\mathrm{C}_{hkj} = \mathrm{upd}\left(\mathbf{has}\,j\ y, \mathrm{C}_{kj}\right)$. Unlike Example 2.25 however, no absent individual exception is raised because $\mathrm{C}_{hkj} \vdash \mathbf{child}\,k \wedge \mathbf{has}\,j\ k$ so $\mathrm{sel}\ (\lambda x.\mathbf{child}\,x \wedge \mathbf{has}\,j\ x)\ \mathrm{C}_{hkj}$ returns k. Performing this substitution in (2.36):

$$\neg\exists\ (\lambda k.\mathbf{child}\,k \wedge \mathbf{has}\,j\ k \wedge \neg\mathbf{happy}\,k) \wedge \phi\mathrm{C}_{ij} \tag{2.37}$$

No exception being raised corresponds to no presupposition accommodation. The context C_{ij} also simplifies:

$$\mathrm{C}_{ij} = \mathrm{upd}\left(\neg\exists\left(\lambda k.\mathbf{child}\,k \wedge \mathbf{has}\,(\mathrm{sel}(\mathrm{named\ ``John"})\mathrm{C}_{kj})k \wedge \neg\mathbf{happy}\ \left(\mathrm{sel}\ \left(\lambda x.\mathbf{child}\ x \wedge \mathbf{has}\ (\mathrm{sel}(\lambda y.\mathbf{male}\,y \wedge \mathbf{human}\,y)\mathrm{C}_{hkj})x\right)\ \mathrm{C}_{hkj}\right)\right), \mathrm{C}_j\right)$$
$$= \mathrm{upd}\left(\neg\exists\ (\lambda k.\mathbf{child}\,k \wedge \mathbf{has}\,j\ k \wedge \neg\mathbf{happy}\,k)\ , \mathrm{C}_j\right)$$

Substituting the normalized subterm (2.37) back into the main term (2.34) gives the interpretation (2.38) of $\mathbf{S}_{(8)}$ in the discourse **D**:

$$\begin{aligned}\mathrm{dupd}\ \mathbf{D}\ \mathbf{S}_{(8)} &= \lambda\phi.\exists\left(\lambda j.\text{named “John”}j \wedge \mathbf{S}_{(8)}\ \mathrm{C}_j\ \phi\ \texttt{handle}\ \ \ldots\right)\\ &\to_{eval} \lambda\phi.\exists\left(\lambda j.\text{named “John”}j \wedge \neg\exists\left(\lambda k.\mathbf{child}\,k \wedge \mathbf{has}\,j\ k \wedge \neg\mathbf{happy}\,k\right) \wedge \phi\mathrm{C}_{ij}\ \texttt{handle}\ \ \ldots\right)\\ &\to_{eval} \lambda\phi.\exists\left(\lambda j.\text{named “John”}j \wedge \neg\exists\left(\lambda k.\mathbf{child}\,k \wedge \mathbf{has}\,j\ k \wedge \neg\mathbf{happy}\,k\right) \wedge \phi\left(\mathrm{upd}\left(\neg\exists\left(\lambda k.\mathbf{child}\,k \wedge \mathbf{has}\,j\ k \wedge \neg\mathbf{happy}\,k\right)\right), \mathrm{C}_j\right)\right)\end{aligned} \tag{2.38}$$

The original interpretation by Lebedeva is

$$\lambda\phi.\exists\left(\lambda j.\text{named “John”}j \wedge \neg\exists\left(\lambda k.\mathbf{child}\,k \wedge \mathbf{has}\,j\ k \wedge \neg\mathbf{happy}\,k\right) \wedge \phi\mathrm{C}_j\right)$$

with the continuation of the discourse receiving an unupdated context.

To conclude the examples, the interpretations of sentences (7) and (8) in discourse **D** with common knowledge C can be compared side by side to see the presupposition projection:

$$\begin{aligned}
&\text{dupd } \mathbf{D}\ \mathbf{S}_{(7)} \\
&\rightarrow^{*}_{eval} \lambda\phi.\exists\Bigg(\lambda j.\text{named ``John''} j \wedge \exists\Bigg(\lambda k.\mathbf{child}\, k \wedge \mathbf{has}\, j\ k \wedge \neg \left(\mathbf{back}\, j \wedge \neg\mathbf{happy}\, k\right) \\
&\quad \wedge \phi\Bigg(\text{upd}\Big(\neg \left(\mathbf{back}\, j \wedge \neg\mathbf{happy}\, k\right), \\
&\qquad (\text{upd}(\mathbf{child}\, k \wedge \mathbf{has}\, j\ k, \text{upd}(\text{named ``John''} j, \text{C})))\Big)\Bigg)\Bigg)\Bigg)
\end{aligned} \tag{2.33}$$

$$\begin{aligned}
&\text{dupd } \mathbf{D}\ \mathbf{S}_{(8)} \\
&\rightarrow^{*}_{eval} \lambda\phi.\exists\Bigg(\lambda j.\text{named ``John''} j \wedge \neg\exists \left(\lambda k.\mathbf{child}\, k \wedge \mathbf{has}\, j\ k \wedge \neg\mathbf{happy}\, k\right) \wedge \\
&\quad \phi\Bigg(\text{upd}\Big(\neg\exists \left(\lambda k.\mathbf{child}\, k \wedge \mathbf{has}\, j\ k \wedge \neg\mathbf{happy}\, k\right), (\text{upd}(\text{named ``John''} j, \text{C}))\Big)\Bigg)\Bigg)
\end{aligned} \tag{2.38}$$

As desired, the presupposition *John has a child*, given by the term $\exists(\lambda k.\mathbf{child}\, k \wedge \mathbf{has}\, j\ k)$, is in the context passed to the continuation of the discourse in term (2.33) but not in (2.38). This captures the observed presupposition projection past the conditional in (7) and the presupposition cancellation in (8).

2.4 Summary

This chapter discussed the merits of a formal semantics for natural language and detailed the formalisms on which this thesis will be based. Each step in the progression from Montague semantics to de Groote's continuation-based dynamic semantics to Lebedeva's systematization and extension was shown to capture more natural language phenomena; the goal of this thesis is to continue in this way.

Chapter 3

Conversational Implicatures

To describe it by what it is not, implicature refers to meaning outside of what is explicitly said, logically entailed or presupposed by an utterance. It is traced back to Frege [1879] and was brought to prominence by Grice [1975], who introduced a provisional division – with prevailing terminology – between *conversational implicature* and *conventional implicature*. For a preliminary distinction, conventional implicatures are associated with particular lexical items, while conversational implicatures are not.

Conversational implicatures are the result of apparent violations of *conversational maxims*. These are principles governing cooperative conversation, such as the *cooperative principle*:

> Make your conversational contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged. (Grice [1975])

Maxims include *be orderly*, *make your contribution as informative as required (but no more than necessary)*, and *be relevant*. Discourse appearing to break these conventions is intuitively interpreted with extra meaning to account for the violation: transgressions are intentional and carry meaning. Grice illustrates with the following discourse:

(11) A. I am out of petrol.

B. There is a garage round the corner.

Supposing B is being cooperative, the implicated meaning is that they believe the garage is open and selling petrol.

A prominent early formalization is by Gazdar [1979], focusing on the reasoning through which implicatures are derived. Of it, Potts [2009] says:

> The specific formal details did not have much influence, owing perhaps to their complexity, but the work remains a touchstone for present-day approaches to presuppositions and conversational implicatures.

Many subsequent treatments of conversational implicatures take the approach of formalizing Grice's maxims directly, using these rules to derive the implicated meaning. According to Beaver [2001]:

> ...formalization of Gricean argumentation is notoriously problematic. Not only are we lacking any generally accepted statement of the Gricean maxims, we are also lacking any generally accepted logic which is able not just to encode those maxims, but also to support the sort of reasoning that would be required.

Sperber and Wilson [1986] simplify the problem of formalizing several maxims to just one in *relevance theory*. This is based on the thesis that conversational implicatures, regardless of their source, can be derived from a single principle of relevance – the notion that every sentence bears some relation to the one preceding.

The current frontier for formalizing conversational implicatures uses different tools – Potts [2006] describes "a shift in emphasis from truth-conditions to probabilities", using probabilistic and game-theoretic approaches to pragmatics. The formalism developed in this chapter is traditional, using only standard tools of logic. This means it cannot capture certain conversational implicature phenomena that other formalisms can, but finds its worth in revealing commonalities between conversational and conventional implicatures – in a fully compositional dynamic semantics. Conversational implicatures are largely absent from dynamic semantic theories because although the formal tools used in dynamic semantics apply readily to a range of context-dependent phenomena, Potts [2009] observes they only serve "to obtain the basic content of a sentence", from which implicature departs.

Of the alternative dynamic semantics to which $GL\chi$ is compared [Lebedeva, 2012, Section 2.2] – Discourse Representation Theory (DRT), Dynamic Predicate Logic (DPL) and Dynamic Montague Grammar (DMG) – only DRT has a treatment of conversational implicatures. Asher and Lascarides [2003]'s Segmented Discourse Representation Theory (SDRT) uses a sentence analysis based on DRT, along with a detailed notion of *discourse structure*, based on divisions into Elementary Discourse Units (EDUs) and relations between them. To illustrate, consider the following examples from Asher and Pogodalla [2010b]:

(12) $\underbrace{\text{We bought the apartment}}_{\pi_1}, \underbrace{\text{but we're renting it}}_{\pi_2}.$

(13) $\underbrace{\text{John fell}}_{\pi_1}. \underbrace{\text{Mary pushed him.}}_{\pi_2}$

The units in (12) are related by the expression $Contrast(\pi_1, \pi_2)$ and the units in (13) are related by $Explanation(\pi_1, \pi_2)$. *Contrast* and *Explanation* are two of many intuitively-named relations whose semantics specify linguistic-semantic properties like *subordinating* or *coordinating* relation – determining availability for anaphora and which other EDUs it may be related (attached) to – and the essence of the relation itself – *Explanation*, for example, includes a notion of *temporal consequence*. [Asher and Pogodalla, 2010b, p.156] describe the role of discourse relations:

> ... [SDRS-formulae of the form $R(\pi_1, \pi_2)$] define a real transition across information states. They act semantically like complex update operators. Their interpretation reflects the special semantic influence that rhetorical relations have on the propositions they connect.

Some discourse relations are triggered by specific lexical items – the contrast relation is encoded in the interpretation of *but*, for example – while others must be inferred based on defeasible reasoning. To achieve this, SDRT has two logics working in tandem – a logic dealing with the content of sentences, and a reasoning logic dealing with the relations between them. The logic for reasoning about attachments between EDUs, called *glue logic*, does not use the content of the sentence directly, as the content is in (at least) first-order logic, leading to problems in the computation of nonmonotonic consequence. Asher [2013] Instead, the glue logic is a quantifier-free description language with two entailment relations, one capturing a notion of defeasible entailment based on Asher's commonsense reasoning (Asher [1990]).

To capture Grice's conversational implicatures, SDRT is equipped with *cognitive modelling*. This takes a simplified model of cognitive states, based on syllogisms relating goals, intentions and beliefs, and uses it to calculate conversational implicatures. Again, it coheres with the theme of achieving the goal without "resorting to the full expressive power of the 'real' logic in which people reason about their cognitive states" [Asher and Lascarides, 2003, p.376].

This chapter proposes a framework for capturing conversational implicatures that differentiates itself from SDRT by not relying on highly specified languages with non-standard semantics. The starting point is Lebedeva [2012]'s suggested further direction for $GL\chi$ of *conversational implicatures by proof-theoretic abduction*. Problems with its implementation via the exception raising and handling mechanism are encountered as the approach is developed. To address this, we extract the use of abductive reasoning from the proof-theoretic implementation and exploit another feature of the framework – the flexible context structure. By adapting Poole [1988, 1989, 1990]'s framework for explanatory and predictive reasoning in classical logic, which captures commonsense reasoning while adhering to the principle of using only standard tools from mathematics, we capture theory or logic of context, within which implicatures can be identified. We conclude with an example to demonstrate the approach.

3.1 Conversational Implicatures by Proof-theoretic Abduction

As a direction for further development of $GL\chi$, Lebedeva [2012] proposes a treatment of conversational implicatures by *proof-theoretic abduction*. This section explains this approach using the following example from Grice [1975], before presenting the formalization.

(14) A. Smith doesn't seem to have a girlfriend these days.

B. He has been paying a lot of visits to New York lately.

Assuming that B is being a cooperative speaker, providing content relevant to A's statement, B's response contains meaning other than that concerning Smith's visits to New York. Suppose A believes that having a girlfriend in a different city is a reason for frequently visiting that city. Then A might take B's statement to mean that Smith has a girlfriend in New York.

This is an instance of *abductive reasoning*, adopting a statement on the basis that it provides an explanation for another statement that is known to be true. Where deduction is the conclusion of Q from P and $P \to Q$, abduction is the conclusion of P from Q and $P \to Q$. Contemporary interest in abduction is attributed to Peirce [1955], who characterizes it as:

> The surprising fact, C, is observed;
> But if A were true, C would be a matter of course,
> Hence, there is reason to suspect that A is true.

It is *defeasible* form of reasoning, non-deductive and open to revision, alongside scientific induction – reasoning that takes several cases of P and Q occurring together to conclude $P \to Q$.

Although logically invalid, abduction is prolific in human reasoning and Hobbs et al. [1993] argue that it is inherent in interpreting sentences in discourse, based on the hypothesis that "it is commonplace that people understand discourse so well because they know so much". To interpret B's remark in (14) requires not just knowledge of the meaning of words but knowledge of the world – specifically that people spend time with their partners and that spending time with someone who lives elsewhere requires visiting them. This knowledge means reasoning occurs when new information is encountered, motivating the use of proofs to capture natural language meaning.

The idea from Hobbs [2004] is to compute implicatures by attempting to prove the logical form of a sentence, taking as axioms formulae corresponding to the current knowledge base. If no proof is found, an explanation is still sought and can be abduced, adding the facts necessary to the knowledge base. These abduced facts correspond to the implicatures of the sentence in the initial knowledge base, thus viewing implicatures "as an abductive move for the sake of achieving the best interpretation" (Hobbs [2004]).

This is incorporated into the dynamic semantics framework developed in Lebedeva [2012] via the definition of a handler for an exception that checks whether a proposition is provable. The use of an exception raising and handling mechanism means it can be used in the compositional computation of meaning. We illustrate the scheme used by Lebedeva with (14). Suppose we have in our context A's utterance, as well as the background knowledge that if someone has a girlfriend who lives in New York, then they visit New York. We can capture this in first-order logic as follows, where $\mathbf{gf}\,(x, y)$ represents x is the girlfriend of y, **live** is the predicate 'lives in New York' and **visit** is the predicate 'visits New York':

$$\mathrm{C} = \{\neg\exists x.\mathbf{gf}\,(x, S), \forall y.((\exists x.\mathbf{gf}\,(x, y) \wedge \mathbf{live}\,(x)) \rightarrow \mathbf{visit}\,(y))\}$$

With this context, what happens in A's mind after B's utterance can be modelled as follows. A checks if B's statement is entailed by A's knowledge base. This amounts to a search for a proof of the logical interpretation of B's utterance from the formulae in the context:

$$\neg\exists x.\mathbf{gf}\,(x, S), \forall y.((\exists x.\mathbf{gf}\,(x, y) \wedge \mathbf{live}\,(x)) \rightarrow \mathbf{visit}\,(y)) \vdash \mathbf{visit}\,(S)$$

A realises that B's statement is not entailed by A's knowledge base when the proof search fails, instead reaching the following partial proof:

$$\dfrac{\dfrac{\dfrac{\vdash \exists x.\mathbf{gf}(x,S) \wedge \mathbf{live}(x) \qquad \dfrac{}{\mathbf{visit}(S) \vdash \mathbf{visit}(S)}}{(\exists x.\mathbf{gf}(x,S) \wedge \mathbf{live}(x)) \rightarrow \mathbf{visit}(S) \vdash \mathbf{visit}(S)} \rightarrow_I}{\forall y.((\exists x.\mathbf{gf}(x,y) \wedge \mathbf{live}(x)) \rightarrow \mathbf{visit}(y)) \vdash \mathbf{visit}(S)} \forall_I}{\neg\exists x.\mathbf{gf}(x,S), \forall y.((\exists x.\mathbf{gf}(x,y) \wedge \mathbf{live}(x)) \rightarrow \mathbf{visit}(y)) \vdash \mathbf{visit}(S)} w$$

The proof search is defined so that an exception is raised when a proof is not found,[1] the handler of which performs abduction of the facts required to complete the proof. In this case, A abduces that Smith has a girlfriend living in New York, and it is propagated down the tree to fix the proof:

$$\dfrac{\dfrac{\dfrac{\dfrac{}{[\exists x.\mathbf{gf}(x,S) \wedge \mathbf{live}(x)] \vdash \exists x.\mathbf{gf}(x,S) \wedge \mathbf{live}(x)} \qquad \dfrac{}{\mathbf{visit}(S) \vdash \mathbf{visit}(S)}}{[\exists x.\mathbf{gf}(x,S) \wedge \mathbf{live}(x)], (\exists x.\mathbf{gf}(x,S) \wedge \mathbf{live}(x)) \rightarrow \mathbf{visit}(S) \vdash \mathbf{visit}(S)} \rightarrow_I}{[\exists x.\mathbf{gf}(x,S) \wedge \mathbf{live}(x)], \forall y.((\exists x.\mathbf{gf}(x,y) \wedge \mathbf{live}(x)) \rightarrow \mathbf{visit}(y)) \vdash \mathbf{visit}(S)} \forall_I}{[\exists x.\mathbf{gf}(x,S) \wedge \mathbf{live}(x)], \ldots, \forall y.((\exists x.\mathbf{gf}(x,y) \wedge \mathbf{live}(x)) \rightarrow \mathbf{visit}(y)) \vdash \mathbf{visit}(S)} w$$

The abduced knowledge is the implicature of B's utterance and is added to A's context. In this case, the update leads to an unsatisfiable context containing both $\exists x.\mathbf{gf}(x,S)$ and $\neg\exists x.\mathbf{gf}(x,S)$. Such a contradiction is defeasible, in the sense that it is the result of defeasible reasoning rather than strictly deductive reasoning, but it must still be corrected. The framework contains a function to check for inconsistencies, raising an exception with a handler that analyses the proof and eliminates propositions to make the knowledge base consistent. A has a choice of which fact to be removed and may proceed by confirming this choice with B.

3.1.1 Formalization

This section presents the formalization of the above analysis from Lebedeva [2012]. The analysis is formalized with a new version of the discourse update function, originally given in Definition 2.24. This includes a new oracle function, checkprovable, which determines whether the proposition is provable from the context in question. If the proposition can be proved the argument is returned, otherwise an exception is raised.

[1] Since provability in first order logic is undecidable, it is not possible to perfectly engineer this behaviour. It would be necessary to require that the proof search algorithm halts, which means that an exception could be raised incorrectly for a provable formula. In advancing the approach of proof-theoretic abduction, it is assumed that the proof search algorithm could be engineered in such a way that this is uncommon.

Definition 3.1 (Provability function)**.**

$$\text{checkprovable } \mathbf{S}\ e\ \phi = \begin{cases} \mathbf{S}\ e\ \phi & \text{if } e \vdash \mathbf{S}e(\lambda e.\top) \\ \text{raise } (\texttt{UnprovablePropExc}(\mathbf{S}e(\lambda e.\top))) & \text{otherwise} \end{cases}$$

The new discourse update function includes two new exception handlers. The first, 3.1d, updates the context that cannot be used to prove the proposition by using an oracle function abd, which abduces facts necessary to complete the proof. This is then checked for consistency by the function con, defined in 3.3. If the context is consistent, evaluation with respect to this new context proceeds. If it is inconsistent, an inconsistent context exception is raised. This is handled in a subsequent gacc call by the handler in 3.1e, which contains an oracle function makesat , which decides which axioms to remove to make the context consistent.

Definition 3.2 (Discourse update function)**.**

$$\text{dupd } \mathbf{D}\ \mathbf{S} := \lambda\phi.\mathbf{D}(\lambda e.\text{gacc } \mathbf{S}\ e\ \phi) \tag{3.1a}$$

$$\text{gacc } \mathbf{S}\ e\ \phi := \Bigg(\Big(\text{checkprovable } \mathbf{S}\ e\ \phi \tag{3.1b}$$

$$\texttt{handle}\ (\texttt{AbsentIndividualExc}\ Q)\ \texttt{with}$$

$$\exists(\lambda x.(Qx) \wedge \text{gacc } \mathbf{S}\ (\text{upd}(Qx, e))\ \phi)\Big) \tag{3.1c}$$

$$\texttt{handle}\ (\texttt{UnprovablePropExc}\ F)\ \texttt{with}$$

$$\text{gacc } \mathbf{S}\ (\text{con}(\text{abd}(F, e)))\ \phi\Bigg) \tag{3.1d}$$

$$\texttt{handle}\ (\texttt{InconsistentContextExc}\ e')\ \texttt{with}$$

$$\text{gacc } \mathbf{S}\ (\text{makesat } e')\ \phi \tag{3.1e}$$

Definition 3.3 (Context consistency function)**.**

$$\text{con } e = \begin{cases} e & \text{if consistent } e \\ \text{raise } (\texttt{InconsistentContextExc}\ e) & \text{otherwise} \end{cases}$$

3.1.2 Simplifying the Dynamization Function

With the introduction of checkprovable as the first function called in gacc, Lebedeva [Lebedeva, 2012, Section 7.2.4] observes that the role of upd is subsumed by the abduction process in the exception handler of `UnprovablePropExc`. Thus the original definition (3.2) for the dynamization of a term $\mathrm{P} : \gamma \to o$ (see Section 2.3.2.1) is replaced by (3.3):

$$\mathbb{D}_o[\mathrm{P}] := \lambda e\phi.\mathrm{P}e \wedge \phi(\mathrm{upd}(\mathrm{P}e, e)) \tag{3.2}$$

$$\mathbb{D}^\star_o[\mathrm{P}] := \lambda e\phi.\mathrm{P}e \wedge \phi e \tag{3.3}$$

It is not obvious that (3.3) performs the same role as (3.2): only the preservation of the formal result of the conservation theorem is remarked on in Lebedeva [2012]. This section demonstrates how the new definition is intended to work by comparing the interpretation of a proposition under the old and new dynamization functions and identifies the unintended consequence that certain dynamic behaviours are lost.

With the static proposition $\mathrm{P} : \gamma \to o$, let its dynamization be $\mathbf{P}^\star = \mathbb{D}^\star_o[\mathrm{P}]$ and interpret $\mathbf{P}^\star$ in the empty context C_0 and empty discourse $\mathbf{D}_0 = \lambda\phi.\phi\mathrm{C}_0$:

$$\begin{aligned} \mathrm{dupd}\ \mathbf{D}_0\ \mathbf{P}^\star &\to^*_\beta \lambda\phi.\mathrm{gacc}\ \mathbf{P}^\star\ \mathrm{C}_0\ \phi && \text{(by (2.25))}\\ &= \lambda\phi.\mathrm{checkprovable}\ \mathbf{P}^\star\ \mathrm{C}_0\ \phi\ \texttt{handle}\ \ldots && \text{(by (3.1b))} \end{aligned}$$

Since C_0 is empty, $\mathrm{C}_0 \nvdash \mathbf{P}^\star\ \mathrm{C}_0\ (\lambda e.\top)$ and so the call to checkprovable returns an exception. This is subsequently handled:

$$\begin{aligned} &\lambda\phi.\mathrm{checkprovable}\ \mathbf{P}^\star\ \mathrm{C}_0\ \phi\ \texttt{handle}\ \ldots\\ &= \lambda\phi.\mathrm{raise}\ (\texttt{UnprovablePropExc}(\mathbf{P}^\star\ \mathrm{C}_0\ (\lambda e.\top)))\ \texttt{handle}\ \ldots && \text{(by 3.1)}\\ &\to_\chi \lambda\phi.\mathrm{gacc}\ \mathbf{P}^\star\ (\mathrm{con}(\mathrm{abd}(\mathbf{P}^\star\ \mathrm{C}_0\ (\lambda e.\top), \mathrm{C}_0)))\ \phi && \text{(by (3.1d))} \end{aligned} \tag{3.4}$$

Suppose the oracle function abd performs the 'trivial' abduction of the formula $\mathbf{P}^\star\ \mathrm{C}_0\ (\lambda e.\top)$ itself, updating the context. The updated context, $\mathrm{C}_1 = \mathrm{upd}(\mathbf{P}^\star\ \mathrm{C}_0\ (\lambda e.\top), \mathrm{C}_0)$, is consistent as it contains only one formula (itself assumed to be consistent). Therefore the evaluation of term 3.4 continues:

$$\begin{aligned} &\lambda\phi.\mathrm{gacc}\ \mathbf{P}^\star\ \mathrm{C}_1\ \phi\\ &= \lambda\phi.\mathrm{checkprovable}\ \mathbf{P}^\star\ \mathrm{C}_1\ \phi\ \texttt{handle}\ \ldots && \text{(by (3.1b))}\\ &= \lambda\phi.\mathbf{P}^\star\ \mathrm{C}_1\ \phi\ \texttt{handle}\ \ldots && \text{(by Definition (3.1))} \end{aligned}$$

Since $\mathrm{C}_1 \vdash \mathbf{P}^\star\ \mathrm{C}_1\ (\lambda e.\top)$, this time checkprovable does not raise an exception. Supposing there are no other exceptions to handle:

$$\begin{aligned}\lambda\phi.\mathbf{P}^\star\ \mathrm{C}_1\ \phi &= \lambda\phi.(\lambda e\phi.\mathrm{P}e \wedge \phi e)\mathrm{C}_1\phi \\ &\rightarrow^*_\beta \lambda\phi.\mathrm{PC}_1 \wedge \phi\mathrm{C}_1 \\ &= \lambda\phi.\mathrm{P}\left(\mathrm{upd}(\mathbf{P}^\star\ \mathrm{C}_0\ (\lambda e.\top), \mathrm{C}_0)\right) \wedge \phi\left(\mathrm{upd}(\mathbf{P}^\star\ \mathrm{C}_0\ (\lambda e.\top), \mathrm{C}_0)\right)\end{aligned} \tag{3.5}$$

and so the continutation of the discourse ϕ is interpreted with respect to the context updated with the content of the sentence, as desired.

To complete the comparison of the original dynamization function and the simplification, we compute the discourse interpretation of a proposition $\mathrm{P} : \gamma \rightarrow o$ under the original dynamization function: $\mathbf{P} = \mathbb{D}_o[\mathrm{P}]$. Interpret $\mathbf{P}$ in the empty context C_0 and empty discourse $\mathbf{D}_0 = \lambda\phi.\phi\mathrm{C}_0$, which proceeds without any exceptions to handle:

$$\begin{aligned}\mathrm{dupd}\ \mathbf{D}_0\ \mathbf{P} &\rightarrow^*_\beta \lambda\phi.\mathrm{gacc}\ \mathbf{P}\ \mathrm{C}_0\ \phi && \text{(by (2.25))} \\ &= \lambda\phi.\mathbf{P}\ \mathrm{C}_0\ \phi\ \mathtt{handle}\ \ldots \\ &= \lambda\phi.\mathbf{P}\ \mathrm{C}_0\ \phi \\ &= \lambda\phi.\left(\lambda e\phi.\mathrm{P}e \wedge \phi(\mathrm{upd}(\mathrm{P}e, e))\right)\mathrm{C}_0\phi \\ &\rightarrow^*_\beta \lambda\phi.\mathrm{PC}_0 \wedge \phi(\mathrm{upd}(\mathrm{PC}_0, \mathrm{C}_0))\end{aligned} \tag{3.6}$$

Table (3.1) shows these terms side by side.

	Sentence interpretation	Discourse interpretation
Old	$\lambda e\phi.\mathrm{P}e \wedge \phi(\mathrm{upd}(\mathrm{P}e, e))$	$\lambda\phi.\mathrm{PC}_0 \wedge \phi(\mathrm{upd}(\mathrm{PC}_0, \mathrm{C}_0))$
New	$\lambda e\phi.\mathrm{P}e \wedge \phi e$	$\lambda\phi.\mathrm{P}\left(\mathrm{upd}(\mathbf{P}\ \mathrm{C}_0\ (\lambda e.\top), \mathrm{C}_0)\right) \wedge \phi\left(\mathrm{upd}(\mathbf{P}\ \mathrm{C}_0\ (\lambda e.\top), \mathrm{C}_0)\right)$

Table 3.1: Comparison of old and new dynamization functions on the level of sentence and discourse interpretations.

Two important differences between the behaviour of the dynamization functions are apparent. Firstly, the old dynamization function updates the context at the sentence level in the subterm $\phi(\mathrm{upd}(\mathrm{P}e, e))$, without requiring a discourse update, while the new dynamization function updates the context at the discourse-level in the subterm $\phi\left(\mathrm{upd}(\mathbf{P}\ \mathrm{C}_0\ (\lambda e.\top), \mathrm{C}_0)\right)$, occuring only with a discourse update.

The significance of this is both technical and linguistic. On the technical side, since the features of GLχ demonstrated with the old dynamization function depend on behaviour on the sentence level, relocating context update to the discourse level means they no longer necessarily hold: properties like accounting for presupposition projection must be re-established. Linguistically, it remains to consider the importance of being able to interpret individual sentences independently of discourse, or whether it is acceptable to sacrifice this for simplicity.

The second important difference is that the new dynamization function interprets the sentence with respect to a context already containing the sentence. This is seen in the subterm $\mathrm{P}\left(\mathrm{upd}(\mathbf{P}\ \mathrm{C}_0\ (\lambda e.\top), \mathrm{C}_0)\right)$ of the new discourse interpretation, interpreting P in a context necessarily containing P, as opposed to the equivalent subterm in the old discourse interpretation PC_0, which does not exhibit this behaviour. Again, the technical ramifications of this remain to be investigated. Regarding intuition about language interpretation, we consider whether the framework should be representative of how people interpret discourse. If it should, then it does not make sense to interpret a fresh sentence in a context in which it is already contained.

Whether features of $GL\chi$ that rely on behaviour on the sentence level are preserved with the simplified dynamization function is now investigated by looking at dynamic conjunction.

3.1.2.1 Dynamic Conjunction Under the Simplified Dynamization Function

In $GL\chi$, *and* is interpreted as the dynamic conjunction connective, in which the second conjunct is interpreted in the context of the first conjunct. To see how this works with the original dynamization function, consider static propositions $\mathrm{P}, \mathrm{Q} : \gamma \to o$ and their dynamizations $\mathbf{P} = \mathbb{D}[\mathrm{P}]$ and $\mathbf{Q} = \mathbb{D}[\mathrm{Q}]$. Computing their conjunction by β-reduction:

$$\begin{aligned}
\mathbf{P} \,\overline{\wedge}\, \mathbf{Q} &= \left(\lambda \mathbf{AB}.\lambda e\phi.\mathbf{A}e(\lambda e.\mathbf{B}e\phi)\right) \mathbf{P}\ \mathbf{Q} \\
&\to_\beta^* \lambda e\phi.\mathbf{P}e(\lambda e.\mathbf{Q}e\phi) \\
&= \lambda e\phi.\left(\lambda e\phi.Pe \wedge \phi\left(\mathrm{upd}(Pe,e)\right)\right) e(\lambda e.\mathbf{Q}e\phi) \\
&\to_\beta^* \lambda e\phi.Pe \wedge \left(\lambda e.\mathbf{Q}e\phi\right)\left(\mathrm{upd}(Pe,e)\right) \\
&\to_\beta \lambda e\phi.Pe \wedge \mathbf{Q}\left(\mathrm{upd}(Pe,e)\right)\phi && \text{(3.7a)} \\
&= \lambda e\phi.Pe \wedge \left(\lambda e\phi.Qe \wedge \phi\left(\mathrm{upd}(Qe,e)\right)\right)\left(\mathrm{upd}(Pe,e)\right)\phi \\
&\to_\beta^* \lambda e\phi.Pe \wedge Q\left(\mathrm{upd}(Pe,e)\right) \wedge \phi\left(\mathrm{upd}\left(Q\left(\mathrm{upd}(Pe,e)\right), \left(\mathrm{upd}(Pe,e)\right)\right)\right) && \text{(3.7b)}
\end{aligned}$$

The term (3.7b) is a logical conjunction of three subterms clearly demonstrating the incremental update of the context within the interpretation of the sentence: the first subterm $\mathrm{P}e$ is the proposition P with respect to the original context, the second subterm $\mathrm{Q}(\mathrm{upd}(\mathrm{P}e, e))$ is the second proposition Q with respect to the context updated with the interpretation of P and the third subterm is the continuation of the discourse evaluated with respect to the context updated initially with the first conjunct and then subsequently with the second conjunct.

This incremental context update is no longer clear using the simplified dynamization function. For static propositions $\mathrm{P}, \mathrm{Q} : \iota \to o$, their dynamizations under the new function are given by $\mathbf{P}^\star = \mathbb{D}^\star(\mathrm{P})$ and $\mathbf{Q}^\star = \mathbb{D}^\star(\mathrm{Q})$ and their conjunction is computed with β-reduction as follows:

$$
\begin{aligned}
\mathbf{P}^\star \mathbin{\overline{\wedge}} \mathbf{Q}^\star &= \big(\lambda \mathbf{A}\mathbf{B}.\lambda e\phi.\mathbf{A}e(\lambda e.\mathbf{B}e\phi)\big)\, \mathbf{P}^\star\mathbf{Q}^\star \\
&\to_\beta^* \lambda e\phi.\mathbf{P}^\star e(\lambda e.\mathbf{Q}^\star e\phi) \\
&= \lambda e\phi.\, \big(\lambda e\phi.\mathrm{P}e \wedge \phi e\big)\, e(\lambda e.\mathbf{Q}^\star e\phi) \\
&\to_\beta^* \lambda e\phi.\mathrm{P}e \wedge (\lambda e.\mathbf{Q}^\star e\phi)e \\
&\to_\beta \lambda e\phi.\mathrm{P}e \wedge \mathbf{Q}^\star e\phi && \text{(3.8a)} \\
&= \lambda e\phi.Pe \wedge \big(\lambda e\phi.Qe \wedge \phi e\big)\, e\phi \\
&\to_\beta \lambda e\phi.Pe \wedge Qe \wedge \phi e && \text{(3.8b)}
\end{aligned}
$$

Comparing term (3.8a) with the equivalent term (3.7a) in the computation with the old dynamization, it is not clear that $\mathbf{Q}^\star$ is being evaluated with respect to a context containing P. Similarly, comparing term (3.8b) with (3.7b), the incremental update of the context within the interpretation of the sentence is absent.

The order of evaluation of the conjuncts is not captured on the sentence level; we proceed by seeing whether the desired behaviour captured on the level of discourse. This raises the same questions as before – is the desired behaviour captured on the level of discourse, and is it acceptable to not capture it on the sentence level? Moreover, the definition of dynamic conjunction appears redundant with the simplification, given how the expression normalizes.

To see what happens on the level of discourse, consider an empty context C_0 and the empty discourse $\mathbf{D}_0 = \lambda\phi.\phi\mathrm{C}_0$. Updating $\mathbf{D}_0$ with the dynamic conjunction of P and Q under the new dynamization function and evaluating, we have:

$$\begin{aligned} \text{dupd}\ \mathbf{D}_0\ \mathbf{P}^\star \mathbin{\overline{\wedge}} \mathbf{Q}^\star &\to^* \lambda\phi.\text{gacc}\ \mathbf{P}^\star \mathbin{\overline{\wedge}} \mathbf{Q}^\star\ \mathrm{C}_0\ \phi && \text{(by (2.25))} \\ &= \lambda\phi.\text{gacc}\ \big(\lambda e\phi.Pe \wedge Qe \wedge \phi e\big)\ \mathrm{C}_0\ \phi \\ &= \lambda\phi.\text{checkprovable}\ \big(\lambda e\phi.Pe \wedge Qe \wedge \phi e\big)\ \mathrm{C}_0\ \phi\ \ \texttt{handle}\ \ldots \end{aligned}$$

Now $(\lambda e\phi.Pe \wedge Qe \wedge \phi e)\mathrm{C}_0(\lambda e.\top)$ reduces to $P\mathrm{C}_0 \wedge Q\mathrm{C}_0$, so inside checkprovable, a proof search for $\mathrm{C}_0 \vdash P\mathrm{C}_0 \wedge Q\mathrm{C}_0$ is performed. Since C_0 is empty, the proof search fails and an exception is raised:

$$\begin{aligned} &\lambda\phi.\text{checkprovable}\ \big(\lambda e\phi.Pe \wedge Qe \wedge \phi e\big)\ \mathrm{C}_0\ \phi\ \ \texttt{handle}\ \ldots \\ &= \lambda\phi.\text{raise}\ (\texttt{UnprovablePropExc}(P\mathrm{C}_0 \wedge Q\mathrm{C}_0))\ \ \texttt{handle}\ \ldots \\ &= \lambda\phi.\text{gacc}\ \big(\lambda e\phi.Pe \wedge Qe \wedge \phi e\big)\ (\text{con}(\text{abd}(P\mathrm{C}_0 \wedge Q\mathrm{C}_0, \mathrm{C}_0)))\ \phi \end{aligned}$$

What happens now depends on the definition of abd. Supposing the logic is classical, P and Q are added to the context, with no specified order of update. In the current form of the proposed simplification of the dynamization function, the desired behaviour of dynamic conjunction as incremental update of the context fails, and the definition of dynamic conjunction is redundant is it behaves like regular conjunction. Therefore the dynamic conjunction with the simplified dynamization function does not exhibit the behaviour that allows phenemena like presupposition projection to be captured.

3.1.3 Considerations for Further Development

In light of these technical issues, this section considers which features of the approach to conversational implicatures by proof-theoretic abduction should be preserved, which should additionally be incorporated and how to achieve this.

Lebedeva's proposal focuses on abduction, rather than common-sense reasoning more broadly. While it demonstrates how abduction can be used to account for implicatures, it omits default reasoning, which can be seen to play a similar role in implicature. Consider again discourse (14):

(14) A: Smith doesn't seem to have a girlfriend these days.
 B: He has been paying a lot of visits to New York lately.

There is a second reasonable interpretation of this discourse involving a different implicature to the original interpretation. Suppose A believes that having a relationship usually means spending time together such that regular trips away prevent Smith from having a girlfriend. Then when A updates their context with B's statement, they reason that B is offering a reason for Smith not having a girlfriend, as opposed to a reason for Smith having a girlfriend. This reasoning depends on a notion of something 'usually being the case', relying on the defaults that relationships usually mean spend time together, and spending lots of time together usually means living in the same city. These are defaults in the sense of not always being the case: for example, it allows exceptions like long-distance relationships. Accounting for conversational implicatures requires capturing implicatures like the second interpretation, and so default reasoning should be enabled alongside abduction.

The details of the abductive mechanism – left unspecified to demonstrate a general concept – inherit the problems of both abduction and proof search. Implementation would require choosing a logic for abduction and a means of proof search while ideally preserving the original principles of the framework, including the use of only standard tools from mathematical logic. These are both non-trivial problems, including specifying how defeasible and non-defeasible information interacts. Furthermore, the abduction mechanism needs to be able to automatically determine whether the formula itself should be trivially abduced, or, in the case of conversational implicatures, the abduction should be another formula from which the proposition in question may be deduced. This is necessary to allow the context update function to be removed – avoiding the redundancy of performing the same context update two different ways – and risks either overgenerating or under generating conversational implicatures.

More fundamentally, there is a sense in which the use of abduction and proof theory have been conflated. That is, it is not *proof-theoretic* abduction that enables conversational implicatures to be captured, meaning another implementation may be better suited.

3.2 Conversational Implicatures by Reasoning in the Context

Based on this, we want a system of abductive and default reasoning with good computational properties and using only familiar tools from logic. Poole's logical framework for default and abductive reasoning (Poole [1988, 1989, 1990]) is a semantics for classical logic that moves from considering reasoning as deduction to reasoning as *theory formation*. This gives certain formulae the status of *hypothesis*, meaning they are prepared to be accepted as part of the theory if they can be used in explanation, but able to be dropped in cases of inconsistency or in favour of a better explanation once new observations are made. This feature of revision in common-sense reasoning tends to make it incompatible with classical logic, which is monotonic: the set of logical consequences is non-decreasing. This means it is either formalized in a non-classical, non-monotonic logic or by augmenting classical logic.

Under Poole's *Theorist* semantics,[2] the problem of non-monotonicity in common-sense reasoning is solved without leaving classical logic. In this way, it is consistent with the philosophy of de Groote's continuation-based dynamic semantics because it uses well-understood, standard tools of logic, within which properties of the formalism may be proved. It also has good computational properties: by design, it is simple to implement (Poole et al. [1987]). The idea is to have a set of facts that the user is not willing to give up, as well as user-provided hypotheses – *defaults* that can be used in prediction and explanation, and *conjectures* that can be used in explanation. Defaults correspond to something *typically* being the case and conjectures correspond to something *possibly* being the case.

This section uses Poole's framework to treat context in $GL\chi$ as a logic with explanation and prediction. To this end, we begin by presenting Poole's framework. Requirements for incorporating a new context structure into $GL\chi$ are established and the framework is adapted and incorporated into $GL\chi$.

3.2.1 Poole's *Theorist* Framework for Default and Abductive Reasoning

Given a standard first order language over a countable alphabet, *formula* refers to a well-formed formula over this language and an *instance of a formula* refers to a substitution of free variables in a formula by terms in the language. The following sets are provided: F of closed formulae, thought of as facts; Δ and Γ of possibly open formulae, constituting the hypotheses of defaults and conjectures respectively; and O of closed formulae of observations about the world. The semantics has three definitions at its core.

Definition 3.4 (Scenarios in *Theorist*)**.** A *scenario* of $(F, \Delta \cup \Gamma)$ is a set $D \cup G$, where D and G are ground instances of elements of Δ and Γ respectively, such that $D \cup G \cup F$ is consistent.

[2] *Theorist* is the name given to the implementation of the semantics; for brevity, we will use this name to refer to both the framework and the implementation.

Definition 3.5 (Explanations in *Theorist*)**.** An *explanation* of a closed formula g from $(F, \Gamma \cup \Delta)$ is a scenario of $(F, \Gamma \cup \Delta)$ that, together with F, implies g.

We follow Poole [1990] in taking our explanations to be least presumptive (not implying other explanations) and minimal (not containing other hypotheses). Adding this requirement means Theorems 6.1 and 6.2 from [Poole, 1989, p.31] apply: if an explanation is minimal or least presumptive then building an explanation incrementally as new propositions are added is the same as building an explanation for the conjunction of those propositions. There may be multiple explanations but based on the examples in Poole [1988, 1989, 1990], which have a similar complexity to the short discourses we are interested in, it is common to have a single explanation and rare to have more than two.

Definition 3.6 (Maximal scenarios)**.** A maximal scenario of (F, Δ) is a maximal set D of ground instances of elements of Δ such that $D \cup F$ is consistent. The set of all maximal scenarios of (F, Δ) is denoted $\max(F, \Delta)$.

Definition 3.7 (Extensions in *Theorist*)**.** An *extension* of (F, Δ) is the logical consequences of a maximal (with respect to set inclusion) scenario of (F, Δ), that is, $Th(F \cup D)$[3] for some maximal set D of ground instances of Δ.

There may be multiple extensions and extensions are infinite.

In Poole [1989], the semantics is given in terms of states.

Definition 3.8 (States in *Theorist*)**.** A *state* of the system is a tuple

$$\langle F, \Delta, \Gamma, O, \mathcal{E} \rangle$$

where, in addition to the sets of formulae defined before, O is a set of closed formulae corresponding to observations and $\mathcal{E}$ is the set of explanations of the observations in O.

To illustrate, consider the following example from Poole [1989] concerning medical diagnosis. Suppose the starting state is $\langle F, \Delta, \Gamma, \{\}, \{\} \rangle$, with sets defined as follows:

$$\begin{aligned} F =& \{\textbf{broken}\,(\text{tibia}) \Rightarrow \textbf{broken}\,(\text{leg})\} \\ \Delta =& \{\textbf{broken}\,(\text{leg}) \Rightarrow \textbf{sore}\,(\text{leg})\} \\ \Gamma =& \{\textbf{broken}\,(\text{leg}), \textbf{broken}\,(\text{tibia})\} \end{aligned}$$

[3] For a set of formulae A, $Th(A)$ is the set of logical consequences of A.

If **sore** (leg) is observed, the new state is $\langle F, \Delta, \Gamma, \{\textbf{sore}\,(\text{leg})\}, \{E_{\text{leg}}\}\rangle$, where E_{leg} is the following explanation:

$$\begin{aligned} E_{\text{leg}} &= D_{\text{leg}} \cup G_{\text{leg}} \\ D_{\text{leg}} &= \{\textbf{broken}\,(\text{leg}) \Rightarrow \textbf{sore}\,(\text{leg})\} \\ G_{\text{leg}} &= \{\textbf{broken}\,(\text{leg})\} \end{aligned}$$

Set D_{leg} is an instantiation of a member of the defaults Δ and set G_{leg} is an instantiation of a conjecture in Γ. Together, they entail – by ordinary modus ponens – the observation **sore** (leg):

$$\textbf{broken}\,(\text{leg}) \Rightarrow \textbf{sore}\,(\text{leg}), \textbf{broken}\,(\text{leg}) \vdash \textbf{sore}\,(\text{leg})$$

Another possible explanation is given by the following set:

$$\begin{aligned} E_{\text{tibia}} &= D_{\text{tibia}} \cup G_{\text{tibia}} \\ D_{\text{tibia}} &= \{\textbf{broken}\,(\text{leg}) \Rightarrow \textbf{sore}\,(\text{leg})\} \\ G_{\text{tibia}} &= \{\textbf{broken}\,(\text{tibia})\} \end{aligned}$$

Along with F, which contains the fact **broken** (tibia) $\Rightarrow$ **broken** (leg), this also entails the observation. It is a minimal explanation, however it is not least presumptive so E_{leg} is preferred.

With this notion of explanation, the default **broken** (leg) $\Rightarrow$ **sore** (leg) acquires the semantics of *having a broken leg is a reason for having a sore leg*. We also wish for it to be interpreted as *having a broken leg usually means having a sore leg*: suppose **broken** (leg) is observed in the initial state, then we wish to predict that the leg is sore. To this end, we proceed by following discussion in [Poole, 1989, Section 2] of how to define prediction.

Consider a stripped-back version of Example 2.1 [Poole, 1989, p.5]:

$$\begin{aligned} F =&\{\forall x.\neg(\textbf{dove}\,(x) \wedge \textbf{hawk}\,(x)), \\ &\textbf{quaker}\,(\text{dick}), \textbf{republican}\,(\text{dick})\} \\ \Delta =&\{\textbf{republican}\,(x) \Rightarrow \textbf{hawk}\,(x) \\ &\textbf{quaker}\,(x) \Rightarrow \textbf{dove}\,(x)\} \end{aligned}$$

Consider the following set:

$$D_1 = \{\textbf{republican}\,(\text{dick}) \Rightarrow \textbf{hawk}\,(\text{dick})\}$$

This is a scenario of (F, Δ) because D_1 is a ground instance of an element of Δ and $D_1 \cup F$ is consistent. There is an explanation for **hawk** (dick) from (F, Δ) because of the following entailment:

$$\textbf{republican}\,(\text{dick}) \in F, (\textbf{republican}\,(\text{dick}) \Rightarrow \textbf{hawk}\,(\text{dick})) \in D_1 \vdash \textbf{hawk}\,(\text{dick})$$

Now consider a different scenario of (F, Δ):

$$D_2 = \{\textbf{quaker}\,(\text{dick}) \Rightarrow \textbf{dove}\,(\text{dick})\}$$

There is an explanation for **dove** (dick) from (F, Δ) using this scenario:

$$\textbf{quaker}\,(\text{dick}) \in F, (\textbf{quaker}\,(\text{dick}) \Rightarrow \textbf{dove}\,(\text{dick})) \in D_2 \vdash \textbf{dove}\,(\text{dick})$$

However, **hawk** (dick) and **dove** (dick) cannot be explained by the same scenario. Such a scenario would be defined in terms of the following set:

$$\begin{aligned} D_3 = \{&\textbf{republican}\,(\text{dick}) \Rightarrow \textbf{hawk}\,(\text{dick}), \\ &\textbf{quaker}\,(\text{dick}) \Rightarrow \textbf{dove}\,(\text{dick})\} \end{aligned}$$

This is not a scenario because $D_3 \cup F$ is inconsistent,with the following contradictory formulae entailed:

$$D_3 \cup F \vdash \neg\textbf{dove}\,(\text{dick}) \wedge \textbf{hawk}\,(\text{dick})$$
$$D_3 \cup F \vdash \textbf{dove}\,(\text{dick}) \wedge \textbf{hawk}\,(\text{dick})$$

Since there is an explanation for both propositions but they cannot both be true, there is the question of what should be predicted. The possibilities are:

(i) either **hawk** (dick) or **dove** (dick), because they are explainable, but not both, because they are inconsistent;

(ii) neither **hawk** (dick) nor **dove** (dick), because **dove** (dick) is evidence against **hawk** (dick) and vice versa;

(iii) their disjunction, because it is true in all extensions and so whichever extension is true in a world, their disjunction will be true in that world; or

(iv) nothing, because there is an inconsistency in the knowledge base and so the knowledge base has an error that needs to be corrected.

We follow Poole [1989] in using the following definition of prediction based on approach (iii).

Definition 3.9 (Predictions in *Theorist*)**.** A formula g is *predicted by* (F, Δ) if g is in every extension of (F, Δ).

Given that there is potentially an infinite number of infinite extensions, Poole describes a dialectical process to identify only "the relevant parts of the relevant extensions" [Poole, 1989, p.27], which is also used to implement prediction. This involves two processes, $\mathcal{Y}$ and $\mathcal{N}$, arguing whether closed formula g should be predicted.

> First $\mathcal{Y}$ tries to find an explanation D of g. Then $\mathcal{N}$ tries to find a scenario inconsistent with D (i.e., an explanation of $\neg D$). $\mathcal{Y}$ must then try to explain g given $\mathcal{N}$'s scenario. ... If $\mathcal{Y}$ cannot come up with an explanation based on $\mathcal{N}$'s scenario, then g is not in all extensions... If $\mathcal{N}$ cannot come up with a scenario inconsistent with all of $\mathcal{Y}$'s arguments, every extension contains at least one of $\mathcal{Y}$'s arguments, and so g is in every extension. [Poole, 1989, p.27]

Where the example is simple enough, we can also determine the predictions by considering the generators of extensions – maximal scenarios of (F, Δ) – as these can be finite.

With the framework defined, we conclude with a reflection on the nature of this semantics:

> ...[the framework] is trying to inherit all of its semantics from the first-order predicate calculus. Semantics is the linking of symbols and sentences in our language with the semantic domain. The semantic domain I am interested in is the real world. This is not a subset of the Herbrand Universe, some Kripke structure or some other mathematical structure (though it may have some relation to such structures), but rather a world consisting of trees and chairs and people and diseases. It is this world that my robot must walk in, and this world that my diagnostic program must reason about to determine what is wrong with a patient. [Poole, 1988, p.4]

3.2.2 *Theorist* for Natural Language

Adapting Theorist for reasoning in natural language interpretation requires categorizing the different kinds of information in a discourse context. Observations, with incremental update as they are encountered, correspond naturally to the content of a discourse. Defaults and conjectures play the same role for this application: this includes both readings of default implication $d_1 \Rightarrow d_2$ (if d_1 is the case then usually d_2 is the case, and d_1 being the case is an explanation for d_2 being the case).[4]

More difficult is what constitutes fact – information that we are not prepared to give up. It is tempting to consider knowledge about the world, such as 'Canberra is in Australia', as fact; however, the intuition in a model-theoretic interpretation is that this knowledge is not necessarily true in every model. In the case of natural language, there is information that must be true in every model: lexical semantic information. Meaning we are not prepared to give up is the meaning of words and relationships between them, the analytic properties of language that hold regardless of context, such as antonymy and 'green is a colour'. Thus we take this to correspond to the facts in Theorist.

There is also the question of where to place individuals and their background information, which is used for anaphora resolution, as in *there exists someone called Ella, Ella is human* and *Ella is female*. Following Lebedeva [2012], to allow discourse referents to be accessible, the existence of individuals is included as previous discourse so that the current discourse is within their scope (as in Example 2.25). Previous discourse is added to the defaults, rather than observations, such that Δ is more accurately described as "defeasible information" – default knowledge and backgrounded knowledge. The difference is that background knowledge is plainly updated – added to Δ without any reasoning occurring in the context. Background information is associated with presupposition phenomena: in Section 4.1 we show how this formalism can be used for capturing presuppositions beyond the referring expressions on which $GL\chi$ focuses.

In addition to these characterizations, Theorist must be expressed in the logic of $GL\chi$. To achieve this, categories of information are formalized as conjunctions of propositions, rather than sets. This is more natural in the setting of λ-calculus and minimises the alterations that need to be made to $GL\chi$ to accommodate reasoning in the context. Unlike Theorist, there are no implicitly universally quantified open

[4]Where the explanatory reading does not make sense, it is possible to formalize the context to exclude it.

formulae: all variables are quantified, although the quantifier may exist outside the context itself in the broader λ-term in which the context is located.

Definition 3.10 (Context logic state in $GL\chi$)**.** A *state* of the discourse context is a tuple of propositions

$$\langle L, \Delta, \Gamma, O, \mathcal{E} \rangle$$

where L is lexical semantic information, O is discourse content, Δ is defeasible information, Γ is conjectures and $\mathcal{E}$ a tuple of propositions corresponding to explanations.

Propositions L and Δ are given by the user to capture the context in which they wish to interpret the discourse. Γ may be given by the user, or automatically generated as the conjunction of antecedents of each implication in Δ. O and $\mathcal{E}$ are initially empty: when O is updated with discourse content, explanations and predictions are computed.

We proceed by defining explanations and predictions in a context, which first requires definitions of ground instance and domain, as well as a specific notion of instance of a context member.

Definition 3.11 (Ground instance in $GL\chi$)**.** A ground instance of a term t in $GL\chi$ is the term that arises by replacing all variables in t by function symbols (including constant symbols).

To transform a term with variables into a ground instance, existential quantifiers are removed by Skolemization, with those outside the scope of universal quantifiers replaced by fresh constants. These constants form the domain of the context. Universal quantifiers are then instantiated by a member of the domain.

Definition 3.12 (Domain of a context in $GL\chi$)**.** For context C and discourse $\mathbf{D}$, the domain of C in $\mathbf{D}$ is the set of fresh constants introduced in the Skolemization of existentially quantified variables of which C is in scope.

This definition works as follows. The presupposition handling mechanism of $GL\chi$ (Definition 2.24a) means that when a new individual is referred to in the discourse, an existentially quantified variable is added to the beginning of the term, with the term in it's scope. This means that the universally quantified formulae in the context are all within the scope of the existentially quantified variables corresponding to discourse referents, which themselves are not in the scope of any universal quantifiers. Then, for example, in the following discourse the domain of context C is $\{j, m\}$:

$$\lambda\phi.\exists\Big(\lambda j.\text{named ``John''}j \wedge \exists\left(\lambda m.\text{named ``Mary''}m \wedge \phi\mathrm{C}\right)\Big)$$

Definition 3.13 (Instance of a context member)**.** Let C be a context in discourse **D** and $\Pi = \pi_1 \wedge \pi_2 \wedge \cdots \wedge \pi_n$ be a member of this context tuple. Let $\overline{\Pi}$ be the set of all possible ground instances, with respect to the domain of C in **D**, of the propositions π_i. Then an instance of Π is a proposition $P = p_1 \wedge \cdots \wedge p_m$ for $\{p_1, \ldots, p_m\}$ a consistent subset of $\overline{\Pi}$. An instance is maximal if it is formed from a maximal consistent subset of $\overline{\Pi}$.

For example, in the above discourse let $\mathrm{C} = \langle L, \Delta, \Gamma, \top, \top\rangle$ with Δ given by:

$$\Delta = \forall(\lambda x.\text{named ``Mary''}x \to \textbf{human}\,x) \wedge \forall(\lambda x.\text{named ``Mary''}x \to \textbf{female}\,x)$$

Then the following are some instances of Δ, with D_4 being a maximal instance:

$$\begin{aligned}
D_1 &= (\text{named ``Mary''}m \to \textbf{human}\,m) \wedge (\text{named ``Mary''}m \to \textbf{female}\,m)\\
D_2 &= (\text{named ``Mary''}j \to \textbf{human}\,j) \wedge (\text{named ``Mary''}j \to \textbf{human}\,j)\\
D_3 &= (\text{named ``Mary''}j \to \textbf{female}\,j)\\
D_4 &= (\text{named ``Mary''}m \to \textbf{human}\,m) \wedge (\text{named ``Mary''}m \to \textbf{female}\,m) \wedge\\
&\qquad (\text{named ``Mary''}j \to \textbf{human}\,j) \wedge (\text{named ``Mary''}j \to \textbf{female}\,j)
\end{aligned}$$

Like Theorist, an explanation of observations in a context logic state in $GL\chi$ is defined in terms of scenarios. However, we make a more general definition of scenario to allow for different combinations of information and generics as follows.

Definition 3.14 (Scenario, generalized)**.** For S and Π members of a context tuple, a scenario of (S, Π) is a proposition P, where P is an instance of Π such that $S \wedge P$ is consistent. A scenario is maximal if P is a maximal instance of Π.

We are interested in scenarios of $(L, \Delta \wedge \Gamma)$ for explanations, as in Theorist, and scenarios of $(O \wedge G, \Delta)$, where G is an instance of Γ, for predictions. We proceed to define this usage.

Definition 3.15 (Explanations in a context)**.** An explanation of O from $(L, \Delta \wedge \Gamma)$ is a scenario $D \wedge G$ of $(L, \Delta \wedge \Gamma)$ that, together with L, entails O.

Prediction in the context in $GL\chi$ varies from prediction in Theorist, which are made with respect to facts and defaults. For the context logic, predictions are distinguished from *lexical semantic consequence*, with predictions coming from the interaction of defeasible information, observations, and the conjectures used in explanation, while lexical semantic consequences referring to entailments of observations and lexical semantic information. For example, if the observation $\textbf{quiet}\, t$ is made in a context where L includes $\forall x.\textbf{quiet}\, x \rightarrow \neg\textbf{loud}\, x$ as a conjunct, then $\neg\textbf{loud}\, t$ is a lexical semantic consequence of the context. It does not make sense to add this consequence as a prediction, since it is the result of a non-defeasible implication.To capture this, we again make a more generic definition of extension.

Definition 3.16 (Extension, generalized)**.** For $S : o$ and $\Pi : o$ members of a context tuple, an extension of (S, Π) is the set of logical consequences of a maximal scenario P_{max} of (S, Π), denoted $Th(S, P_{\text{max}})$.

Definition 3.17 (Predictions)**.** Suppose $D \wedge G$ is an explanation of O. Then proposition t is predicted by $(O \wedge G, \Delta)$ if t is in every extension of $(O \wedge G, \Delta)$.

To determine the predictions of a context requires determining the extensions of $(O \wedge G, \Delta)$. Maximal scenarios of $(O \wedge G, \Delta)$ are the generators of extensions and can be found by considering maximal instances of Δ.

The inclusion of conjectures in the computation of predictions is because if there is reason to believe conjecture a due to default $a \rightarrow b$ and observation b, and in addition $a \rightarrow c$, then we wish to predict c. This is achieved by indexing the prediction sets by the explanations and including the conjectures of the relevant explanation in the set for which extensions are considered; however, if there are no explanations, then $G = \top$ and predictions are made based solely on observations.

Definition 3.18 (Theory of context)**.** The theory of context C refers to both C and the predictions computed from C.

To make a distinction between the theory of context and the former notion of context without defeasible reasoning, one further definition is made.

Definition 3.19 (Implicatures of a context)**.** Suppose a context C has explanations $\mathcal{E} = \langle E_1, \ldots, E_k \rangle$ and corresponding predictions $P_1, \ldots, P_k$. Then the implicatures of C are the explanations and predictions, given by the following set:

$$I(\mathrm{C}) = \{E_1 \wedge P_1, E_2 \wedge P_2, \ldots, E_k \wedge P_k\}$$

When there are no explanations, there is a single prediction based on observations and defaults. In this case, the implicatures of C are just this prediction, that is, $\mathcal{E} = \langle \top \rangle$ and $I(C) = \{\top \wedge P_1\} = \{P_1\}$.

3.2.2.1 Modifications to $GL\chi$

With these definitions in hand, the context update function may be defined. This adds a proposition from the interpretation of a sentence to the context as an observation, computes explanations of the observations and makes predictions.

Definition 3.20 (Context logic update in $GL\chi$)**.** Given a context

$$\mathrm{C}_n = \langle L_n, \Delta_n, \Gamma_n, O_n, \mathcal{E}_n \rangle,$$

context update by a term $t : o$ in $GL\chi$ is given by:

$$\begin{aligned}\mathrm{upd}_O(t, \mathrm{C}_n) &= \mathrm{C}_{n+1} \\ &:= \langle L_n, \Delta_n, \Gamma_n, O_{n+1}, \mathcal{E}_{n+1} \rangle\end{aligned}$$

with updates defined as follows:

$$O_{n+1} = O_n \wedge t \tag{3.9a}$$

$$\mathcal{E}_{n+1} = \langle E_1, \ldots, E_m \rangle \tag{3.9b}$$

$$\text{where } E_i = D_i \wedge G_i \text{ is an explanation of } O_{n+1}$$

If there are no explanations of O_{n+1}, let $\mathcal{E}_{n+1} = \top$.

Equation (3.9a) adds the proposition t to the set of observations. Equation (3.9b) specifies the set of explanations of the updated observations. Where there are multiple explanations, there is – informally – a preference for the least presumptive and minimal explanation.

The above definition of context update replaces Definition 2.23, which performed context update for context as a conjunction of propositions. We proceed in this way by modifying the other definitions in Section 2.3.4 that are not generic, instead depending on the context structure.

Firstly, equation (2.7b) of the dynamization function from Definition 2.10 is tweaked to define it in terms of the new context theory update function:

$$\mathbb{D}[\mathrm{P}] := \lambda e\phi.\mathrm{P}e \wedge \phi(\mathrm{upd}_O(\mathrm{P}e, e))$$

Definition 2.22 of the special constant sel is updated, specifying that individuals are chosen with respect to defeasible information, observations and lexical semantic information.

Definition 3.21 (Selection function with context logic)**.** Let $\mathrm{C} = \langle L, \Delta, \Gamma, O, \mathcal{E} \rangle$, then:

$$\mathrm{sel}\ P\ \mathrm{C} := \begin{cases} \mathrm{choose}\ \{\mathtt{a} \mid \{L \wedge \Delta \wedge O \vdash P\mathtt{a}\} & \text{if } \{L \wedge \Delta \wedge O \vdash P\mathtt{a}\} \neq \emptyset \\ \mathrm{raise}\ (\mathtt{AbsentIndividualExc}\ P) & \text{otherwise} \end{cases}$$

Finally, the discourse update function from Definition 2.24 is tweaked to specify that in the handler for the absent individual exception, the new referent is added to the defeasible information of the context. This is because presupposition accommodation accounts for information assumed by the speaker to be in the common ground, as opposed to at-issue content, which is added to the observations. For this adjustment, a notion of background update is defined.

Definition 3.22 (Background update)**.** Let $\mathrm{C} = \langle L, \Delta, \Gamma, O, \mathcal{E} \rangle$ and $t : o$, then background update is defined

$$\mathrm{upd}_\Delta(t, \mathrm{C}) := \langle L, \Delta \wedge t, \Gamma, O, \mathcal{E} \rangle$$

Definition 3.23 (Discourse update function with elaborated context structure)**.** For $\mathbf{D} : (\gamma \rightarrow o) \rightarrow o$ and $\mathbf{S} : \gamma \rightarrow (\gamma \rightarrow o) \rightarrow o$,

$$\begin{aligned} \mathrm{dupd}\ \mathbf{D}\ \mathbf{S} &:= \lambda\phi.\mathbf{D}(\lambda e.\mathrm{gacc}\ \mathbf{S}\ e\ \phi) \\ \mathrm{gacc}\ \mathbf{S}\ e\ \phi &:= \mathbf{S}\ e\ \phi \\ &\quad \mathtt{handle}\ (\mathtt{AbsentIndividualExc}\ \mathrm{P})\ \mathtt{with} \\ &\quad \exists(\lambda x.(\mathrm{P}x) \wedge \mathrm{gacc}\ \mathbf{S}\ (\mathrm{upd}_\Delta(\mathrm{P}x, e))\ \phi) \end{aligned}$$

3.2.3 Examples

The new context structure is demonstrated on discourse (14). As the focus is on discourse interpretation, certain details of obtaining the sentence form compositionally are omitted: the strength of the framework in doing this has already been well-demonstrated. Two reasonable interpretations were identified; here we show how both can be captured using reasoning in the context.

Example 3.24 (*Smith doesn't seem to have a girlfriend these days*, sentence interpretation)**.** Beyond the interpretations of various lexical items given in Lebedeva [2012], interpreting (14) requires an interpretation of the modal verb *to seem*. Capturing modality is beyond the scope of this thesis; all that is required is to differentiate the meaning of (14)a from "Smith doesn't have a girlfriend". We assign the following interpretation to *seem*:

$$\overline{[\![seem]\!]} = \lambda\mathbf{P}.\left(\lambda e\phi.\mathbf{seem}\ (\mathbf{P}e(\lambda e.\top)) \wedge \phi\left(\mathrm{upd}\left(\mathbf{seem}\ (\mathbf{P}e(\lambda e.\top))\,,e\right)\right)\right)$$

where $\mathbf{P}$ is a proposition. The sentence-level interpretation is given by the following composition of lexical items:

$$\mathbf{S}_{(14)a} = \widetilde{[\![not]\!]}\left(\overline{[\![seem]\!]}\left(\overline{[\![has]\!]}\left(\widetilde{[\![a]\!]}\ \overline{[\![girlfriend]\!]}\right)\overline{[\![Smith]\!]}\right)\right)$$

for which the interpretations of individual lexical items are:[5]

$$\begin{aligned}
\widetilde{[\![a]\!]} &= \lambda\mathbf{PQ}.\overline{\exists}(\lambda\mathbf{x}.\mathbf{Px}\ \overline{\wedge}\ \mathbf{Qx}) \\
\overline{[\![girlfriend]\!]} &= \overline{\mathbf{girlfriend}} \\
\overline{[\![has]\!]} &= \lambda\mathbf{YX}.\mathbf{X}\left(\lambda\mathbf{x}.\mathbf{Y}(\lambda\mathbf{y}.\overline{\mathbf{has}}\,\mathbf{xy})\right) \\
\widetilde{[\![Smith]\!]} &= \lambda\mathbf{P}.\mathbf{P}(\mathrm{sel}(\text{named "Smith"}))
\end{aligned}$$

Composing these meanings:

$$\begin{aligned}
&\overline{[\![has]\!]}\ (\widetilde{[\![a]\!]}\ \overline{[\![girlfriend]\!]}\ )\widetilde{[\![Smith]\!]} \\
&\to^*_\beta \lambda e\phi.\exists\Bigg(\lambda y.\mathbf{girlfriend}\,y \wedge \mathbf{has}\left(\mathrm{sel}(\text{named "Smith"})(\mathrm{upd}_O(\mathbf{girlfriend}\,y,e))\right)\,y \\
&\qquad \wedge \phi\left(\mathrm{upd}_O\left(\mathbf{has}\left(\mathrm{sel}(\text{named "Smith"})(\mathrm{upd}_O(\mathbf{girlfriend}\,y,e))\right)\,y, \mathrm{upd}_O(\mathbf{girlfriend}\,y,e)\right)\right)\Bigg)
\end{aligned}$$

[5] See [Lebedeva, 2012, Section 5.1.3] for more on the interpretation of the indefinite article.

$$\overline{[\![seem]\!]}\left(\overline{[\![has]\!]}\left(\widetilde{[\![a]\!]}\,\overline{[\![girlfriend]\!]}\right)\overline{[\![Smith]\!]}\right)\to^*_\beta$$

$$\lambda e\phi.\textbf{seem}\left(\exists\left(\lambda y.\textbf{girlfriend}\,y\wedge\textbf{has}\left(\text{sel}(\text{named “Smith”})(\text{upd}_O(\textbf{girlfriend}\,y,e))\right)y\right)\right)$$
$$\wedge\,\phi\left(\text{upd}_O\left(\textbf{seem}\left(\exists\left(\lambda y.\textbf{girlfriend}\,y\wedge\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left(\textbf{has}\left(\text{sel}(\text{named “Smith”})(\text{upd}_O(\textbf{girlfriend}\,y,e))\right)y\right)\right)\right),e\right)\right)$$

Applying the negation to get the final interpretation:[6]

$$\mathbf{S}_{(14)a}\to^*_\beta$$
$$\lambda e\phi.\neg\textbf{seem}\left(\exists\left(\lambda y.\textbf{girlfriend}\,y\wedge\textbf{has}\left(\text{sel}(\text{named “Smith”})(\text{upd}_O(\textbf{girlfriend}\,y,e))\right)y\right)\right)$$
$$\wedge\,\phi\left(\text{upd}_O\left(\neg\textbf{seem}\left(\exists\left(\lambda y.\textbf{girlfriend}\,y\wedge\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left(\textbf{has}\left(\text{sel}(\text{named “Smith”})(\text{upd}_O(\textbf{girlfriend}\,y,e))\right)y\right)\right)\right),e\right)\right)$$

The sentence-level interpretation of utterance (14)b is:

$$\mathbf{S}_{(14)b}\to^*_\beta\lambda e\phi.\textbf{visit}\,\left(\text{sel}(\lambda x.\text{named “Smith”}x)e\right)\,\left(\text{sel}(\lambda x.\text{named “New York”}x)e\right)$$
$$\wedge\phi\left(\text{upd}_O\left(\textbf{visit}\,\left(\text{sel}(\lambda x.\text{named “Smith”}x)e\right)\,\left(\text{sel}(\lambda x.\text{named “New York”}x)e\right),e\right)\right)$$

Example 3.25 (*Smith doesn't seem to have a girlfriend these days* discourse context)**.** The relevant lexical semantic information for interpreting this discourse captures the meaning of *living apart*, that is, *if two people live apart and one of them lives in place Z then the other does not live in place Z*, formalized in the following proposition:

$$L=\forall(\lambda x.\forall(\lambda y.\forall(\lambda z.\textbf{live-apart}\,x\;y\;z\wedge\textbf{live}\,y\;z\to\neg\textbf{live}\,x\;z)))$$

[6]There are two plausible interpretations of the scope of the negation in *Smith doesn't seem to have a girlfriend these days*: it is not the case that Smith seems to have a girlfriend these days, or Smith seems to not have a girlfriend these days. The difference is relevant if the modal meaning of *to seem* is being captured, which is beyond the scope of this thesis.

The relevant default information is that *having a girlfriend who lives elsewhere is a reason for (and usually means) not appearing to have a girlfriend, not having a girlfriend is a reason for (and usually means) not appearing to have a girlfriend* and *having a girlfriend who lives elsewhere is a reason for (and usually means) visiting where they live*. For the selection of pronoun referents, there is also the information that being called Smith usually means being human and being male. This is formalized as the following proposition:

$$
\begin{aligned}
\Delta =&\forall(\lambda x.\text{named “Smith”}x \to \mathbf{human}\, x)\wedge\\
&\forall(\lambda x.\text{named “Smith”}x \to \mathbf{male}\, x)\wedge\\
&\forall\Big(\lambda x.\forall\Big(\lambda y.\Big(\exists(\lambda z.\mathbf{girlfriend}\, z \wedge \mathbf{has}\, x\; z \wedge \mathbf{live\text{-}apart}\, x\; z\; y)\Big)\\
&\qquad \to \neg\mathbf{seem}\,\Big(\exists(\lambda z.\mathbf{girlfriend}\, z \wedge \mathbf{has}\, x\; z)\Big)\Big)\Big)\wedge\\
&\forall\Big(\lambda x.\forall\Big(\lambda y.\Big(\neg(\exists(\lambda z.\mathbf{girlfriend}\, z \wedge \mathbf{has}\, x\; z))\Big)\\
&\qquad \to \neg\mathbf{seem}\,\Big(\exists(\lambda z.\mathbf{girlfriend}\, z \wedge \mathbf{has}\, x\; z)\Big)\Big)\Big)\wedge\\
&\forall\Big(\lambda x.\forall\Big(\lambda y.\Big(\exists(\lambda z.\mathbf{girlfriend}\, z \wedge \mathbf{has}\, x\; z \wedge \mathbf{live\text{-}apart}\, x\; z\; y)\Big) \to \mathbf{visit}\, x\; y)\Big)\Big)
\end{aligned}
$$

The relevant conjectures are that *it is possible to have a girlfriend from whom one lives apart* and *it is possible to not have a girlfriend*:

$$
\begin{aligned}
\Gamma =&\forall\Big(\lambda x.\forall\Big(\lambda y.\Big(\exists(\lambda z.\mathbf{girlfriend}\, z \wedge \mathbf{has}\, x\; z \wedge \mathbf{live\text{-}apart}\, x\; z\; y)\Big)\Big)\Big)\wedge\\
&\forall\Big(\lambda x.\Big(\neg\,(\exists(\lambda z.\mathbf{girlfriend}\, z \wedge \mathbf{has}\, x\; z))\,\Big)\Big)
\end{aligned}
$$

The context is then given by the tuple $\mathrm{C} = \langle L, \Delta, \Gamma, \top, \top\rangle$.

Example 3.26 (*Smith doesn't seem to have a girlfriend these days* discourse, first interpretation)**.** The sentence is to be interpreted with respect to context C in the following discourse:

$$
\begin{aligned}
&\mathbf{D}_0 = \lambda\phi.\exists\Bigg(\lambda s.\text{named “Smith”}s \wedge \exists\Bigg(\lambda n.\text{named “New York”}n \wedge \neg\mathbf{live}\, s\; n\, \wedge\\
&\quad \phi\,\Big(\mathrm{upd}_\Delta\,\Big(\mathbf{lives}\, s\; n, \mathrm{upd}_\Delta\,\big(\text{named “New York”}n, \mathrm{upd}_\Delta(\text{named “Smith”}s, \mathrm{C})\big)\Big)\Big)\Bigg)\Bigg)
\end{aligned}
$$

The update of C with the preceding discourse is abbreviated as the zeroth context, with respect to which the upcoming discourse will be interpreted:

$$\begin{aligned}\mathrm{C}_0 &:= \mathrm{upd}_\Delta\left(\mathbf{lives}\, s\ n, \mathrm{upd}_\Delta\left(\text{named “New York”}n, \mathrm{upd}_\Delta(\text{named “Smith”}s, \mathrm{C})\right)\right)\\ &= \langle L, \Delta \wedge \text{named “Smith”}s \wedge \text{named “New York”}n \wedge \neg\mathbf{live}\, s\ n, \Gamma, \top, \top\rangle\end{aligned}$$

The previous discourse is backgrounded and so there is no reasoning in the context with this update.

The update of $\mathbf{D}_0$ with (14)a is computed as follows:

$$\begin{aligned}&\mathrm{dupd}\ \mathbf{D}_0\ \mathbf{S}_{(14)a}\\ &= \lambda\phi.\mathbf{D}_0(\lambda e.\mathrm{gacc}\ \mathbf{S}_{(14)a}\ e\ \phi)\\ &= \lambda\phi.\Bigg(\lambda\phi.\exists\Big(\lambda s.\text{named “Smith”}s \wedge \exists\Big(\lambda n.\text{named “New York”}n \wedge \neg\mathbf{live}\, s\ n \wedge\\ &\qquad \phi\mathrm{C}_0\Big)\Big)\Bigg)(\lambda e.\mathrm{gacc}\ \mathbf{S}_{(14)a}\ e\ \phi)\\ &\to_\beta \lambda\phi.\exists\Big(\lambda s.\text{named “Smith”}s \wedge \exists\Big(\lambda n.\text{named “New York”}n \wedge \neg\mathbf{live}\, s\ n \wedge\\ &\qquad (\lambda e.\mathrm{gacc}\ \mathbf{S}_{(14)a}\ e\ \phi)\mathrm{C}_0\Big)\Big)\\ &\to_\beta \lambda\phi.\exists\Big(\lambda s.\text{named “Smith”}s \wedge \exists\Big(\lambda n.\text{named “New York”}n \wedge \neg\mathbf{live}\, s\ n \wedge\\ &\qquad \mathrm{gacc}\ \mathbf{S}_{(14)a}\ \mathrm{C}_0\ \phi\Big)\Big)\end{aligned}$$

The computation proceeds in the subterm $\mathbf{S}_{(14)a}\ \mathrm{C}_0\ \phi$ with β-reduction and evaluation of the selection functions:

$$\begin{aligned}
&\mathbf{S}_{(14)a}\ \mathrm{C}_0\ \phi \\
&= \lambda e\phi.\neg\mathbf{seem}\ \Big(\exists\ \Big(\lambda y.\mathbf{girlfriend}\,y \wedge \mathbf{has}\ \Big(\mathrm{sel}(\text{named “Smith”}s)(\mathrm{upd}_O(\mathbf{girlfriend}\,y, e))\Big)\ y\Big)\Big) \\
&\quad \wedge \phi \Bigg(\mathrm{upd}_O\Bigg(\neg\mathbf{seem}\ \Big(\exists\Big(\lambda y.\mathbf{girlfriend}\,y \wedge \\
&\qquad \Big(\mathbf{has}\ \Big(\mathrm{sel}(\text{named “Smith”})(\mathrm{upd}_O(\mathbf{girlfriend}\,y, e))\Big)\ y\Big)\Big)\Big), e\Bigg)\Bigg)\ \mathrm{C}_0\ \phi \\
&\rightarrow_\beta^* \neg\mathbf{seem}\ \Big(\exists\ \Big(\lambda y.\mathbf{girlfriend}\,y \wedge \mathbf{has}\ \Big(\mathrm{sel}(\text{named “Smith”}s)(\mathrm{upd}_O(\mathbf{girlfriend}\,y, \mathrm{C}_0))\Big)\ y\Big)\Big) \\
&\quad \wedge \phi \Bigg(\mathrm{upd}_O\Bigg(\neg\mathbf{seem}\ \Big(\exists\Big(\lambda y.\mathbf{girlfriend}\,y \wedge \\
&\qquad \Big(\mathbf{has}\ \Big(\mathrm{sel}(\text{named “Smith”})(\mathrm{upd}_O(\mathbf{girlfriend}\,y, \mathrm{C}_0))\Big)\ y\Big)\Big)\Big), \mathrm{C}_0\Bigg)\Bigg) \\
&= \neg\mathbf{seem}\ (\exists(\lambda y.\mathbf{girlfriend}\,y \wedge \mathbf{has}\,s\ y)) \\
&\quad \wedge \phi \Big(\mathrm{upd}_O\Big(\neg\mathbf{seem}\ (\exists(\lambda y.\mathbf{girlfriend}\,y \wedge \mathbf{has}\,s\ y)), \mathrm{C}_0\Big)\Big)
\end{aligned}$$

Looking at the context update from C_0 to C_1:

$$\begin{aligned}
\mathrm{C}_1 &= \mathrm{upd}_O\Big(\neg\mathbf{seem}\ (\exists(\lambda y.\mathbf{girlfriend}\,y \wedge \mathbf{has}\,s\ y)), \mathrm{C}_0\Big) \\
&= \langle L_0, \Delta_0, \Gamma_0, \neg\mathbf{seem}\ (\exists\lambda y.\mathbf{girlfriend}\,y \wedge \mathbf{has}\,s\ y), \langle E_{\text{single}}, E_{\text{apart}}\rangle\rangle
\end{aligned}$$

where E_{single} and E_{apart} are computed according to Definition 3.15 as follows. For E_{single}:

$$\begin{aligned}
E_{\text{single}} &= D_{\text{single}} \wedge G_{\text{single}} \\
D_{\text{single}} &= \neg\exists(\lambda y.\mathbf{girlfriend}\,y \wedge \mathbf{has}\,s\ y) \rightarrow \neg\mathbf{seem}\ (\exists(\lambda y.\mathbf{girlfriend}\,y \wedge \mathbf{has}\,s\ y)) \\
G_{\text{single}} &= \neg\exists(\lambda y.\mathbf{girlfriend}\,y \wedge \mathbf{has}\,s\ y)
\end{aligned}$$

since $L \wedge D_{\text{single}} \wedge G_{\text{single}}$ is consistent and

$$L \wedge D_{\text{single}} \wedge G_{\text{single}} \vdash \neg\mathbf{seem}\ (\exists(\lambda y.\mathbf{girlfriend}\,y \wedge \mathbf{has}\,s\ y))$$

This is the explanation that Smith does not appear to have a girlfriend because he is single. For E_{apart}:

$$\begin{aligned}
E_{\text{apart}} &= D_{\text{apart}} \wedge G_{\text{apart}} \\
D_{\text{apart}} &= \exists(\lambda y.\mathbf{girlfriend}\, y \wedge \mathbf{has}\, s\ y \wedge \mathbf{lives\text{-}apart}\, s\ y\ n) \\
&\quad \rightarrow \neg\mathbf{seem}\ \big(\exists(\lambda y.\mathbf{girlfriend}\, y \wedge \mathbf{has}\, x\ y)\big) \\
G_{\text{apart}} &= \exists(\lambda y.\mathbf{girlfriend}\, y \wedge \mathbf{has}\, s\ y \wedge \mathbf{lives\text{-}apart}\, s\ y\ n)
\end{aligned}$$

since $L \wedge D_{\text{apart}} \wedge G_{\text{apart}}$ is consistent and

$$L \wedge D_{\text{apart}} \wedge G_{\text{apart}} \vdash \neg\mathbf{seem}\ \big(\exists(\lambda y.\mathbf{girlfriend}\, y \wedge \mathbf{has}\, s\ y)\big)$$

This is the explanation that Smith does not appear to have a girlfriend because his girlfriend lives elsewhere.

Remark 3.27. Given that there are multiple explanations, this utterance can be viewed as motivating a follow-up from B to clarify: different explanations correspond to different options for continuing the discourse in a relevant way. Adopting the hypothesis that implicatures correspond to explanations and predictions in the context, this utterance can be viewed as implicating either Smith doesn't have a girlfriend[7] or that Smith has a girlfriend living elsewhere. Since the former explanation is less presumptive, it is preferred.

Predictions are found by looking at the extensions of the context, which are defined with respect to maximal instances of Δ. There is only one maximal instance of

[7] This reading warrants following up as it has implications for the meaning of *seems*.

Δ_1, given as follows, where $A = \{s, n\}$ is the domain:

$$
\begin{aligned}
D_{\max} = {} & \wedge_{a\in A}\Big(\forall(\lambda x.\text{named "Smith"}x \to \textbf{human}\,x)\Big)[x/a]\wedge \\
& \wedge_{a\in A}\Big(\forall(\lambda x.\text{named "Smith"}x \to \textbf{male}\,x)\Big)[x/a]\wedge \\
& \wedge_{a\in A}\left(\wedge_{a\in A}\left(\forall\Big(\lambda x.\forall\Big(\lambda y.\Big(\exists(\lambda z.\textbf{girlfriend}\,z \wedge \textbf{has}\,x\,z \wedge \textbf{live-apart}\,x\,z\,y)\Big)\right.\right. \\
& \quad \left.\left.\to \neg\textbf{seem}\Big(\exists(\lambda z.\textbf{girlfriend}\,z \wedge \textbf{has}\,x\,z)\Big)\Big)\Big)\right)[x/a]\right)[y/a]\wedge \\
& \wedge_{a\in A}\left(\wedge_{a\in A}\left(\forall\Big(\lambda x.\forall\Big(\lambda y.\Big(\neg(\exists(\lambda z.\textbf{girlfriend}\,z \wedge \textbf{has}\,x\,z))\Big)\right.\right. \\
& \quad \left.\left.\to \neg\textbf{seem}\Big(\exists(\lambda z.\textbf{girlfriend}\,z \wedge \textbf{has}\,x\,z)\Big)\Big)\Big)\right)[x/a]\right)[y/a]\wedge \\
& \wedge_{a\in A}\left(\wedge_{a\in A}\left(\forall\Big(\lambda x.\forall\Big(\lambda y.\Big(\exists(\lambda z.\textbf{girlfriend}\,z \wedge \textbf{has}\,x\,z \wedge \textbf{live-apart}\,x\,z\,y)\Big)\right.\right. \\
& \quad \left.\left.\to \textbf{visit}\,x\,y)\Big)\Big)\right)[x/a]\right)[y/a]\wedge \\
& \text{named "Smith"}s \wedge \text{named "New York"}n \wedge \textbf{live}\,s\,n
\end{aligned}
$$

The extensions associated with explanation E_{single} are generated by maximal scenarios of $(O_1 \wedge G_{\text{single}}, \Delta_1)$. Since there is only one maximal instance of Δ_1, there is only one such maximal scenario:

$$
\begin{aligned}
O_1 \wedge G_{\text{single}} \wedge D_{\max} = {} & \neg\textbf{seem}\ \big(\exists\lambda y.\textbf{girlfriend}\,y \wedge \textbf{has}\,s\,y\big) \wedge \\
& \neg\exists(\lambda y.\textbf{girlfriend}\,y \wedge \textbf{has}\,s\,y) \wedge D_{\max}
\end{aligned}
$$

This generates the only extension $Th(O_1 \wedge G_{\text{single}}, D_{\max})$, which does not include any non-trivial predictions, where a 'trivial' prediction is, for example, a disjunction of propositions.

The extensions associated with explanation E_{apart} are generated by maximal scenarios of $(O_1 \wedge G_{\text{apart}}, \Delta)$. Since there is only one maximal instantiation of Δ_1, there is only one such maximal scenario:

$$O_1 \wedge G_{\text{apart}} \wedge D_{\max} = \neg\textbf{seem}\ \big(\exists\lambda y.\textbf{girlfriend}\, y \wedge \textbf{has}\, s\ y\big) \wedge \\ \exists(\lambda y.\textbf{girlfriend}\, y \wedge \textbf{has}\, s\ y \wedge \textbf{lives-apart}\, s\ y\ n) \wedge D_{\max}$$

This generates the only extension $Th(O_1 \wedge G_{\text{apart}}, D_{\max})$, which includes the proposition $\textbf{visit}\, s\ n$ since

$$D_{\max} \vdash \exists(\lambda y.\textbf{girlfriend}\, y \wedge \textbf{has}\, s\ y \wedge \textbf{lives-apart}\, s\ y\ n) \rightarrow \textbf{visit}\, s\ n$$

and

$$\exists(\lambda y.\textbf{girlfriend}\, y \wedge \textbf{has}\, s\ y \wedge \textbf{lives-apart}\, s\ y\ n), \\ \exists(\lambda y.\textbf{girlfriend}\, y \wedge \textbf{has}\, s\ y \wedge \textbf{lives-apart}\, s\ y\ n) \rightarrow \textbf{visit}\, s\ n \vdash \textbf{visit}\, s\ n$$

This is predicted in every extension based on explanation E_{apart} and so it is predicted by the context with respect to E_{apart} that Smith visits New York.

Returning to the main discourse term:

$$\mathbf{D}_1 \rightarrow_{eval} \lambda\phi.\exists\Big(\lambda s.\text{named ``Smith''} s \wedge \exists\Big(\lambda n.\text{named ``New York''} n \wedge \neg\textbf{live}\, s\ n \wedge \\ \neg\textbf{seem}\ \big(\exists\lambda y.\textbf{girlfriend}\, y \wedge \textbf{has}\, s\ y\big) \wedge \phi \mathrm{C}_1\Big)\Big)$$

Updating the discourse with the next sentence:

$$
\begin{aligned}
\mathbf{D}_2 &:= \text{dupd } \mathbf{D}_1 \ \mathbf{S}_{(14)b} \\
&= \lambda\phi.\mathbf{D}_1 \left(\lambda e.\text{gacc } \mathbf{S}_{(14)b} \ e \ \phi\right) \\
&= \lambda\phi.\Bigg(\lambda\phi.\exists\Big(\lambda s.\text{named "Smith"} s \wedge \exists\Big(\lambda n.\text{named "New York"} n \wedge \neg\mathbf{live}\, s \ n \ \wedge \\
&\qquad \neg\mathbf{seem}\ \left(\exists\lambda y.\mathbf{girlfriend}\, y \wedge \mathbf{has}\, s \ y\right) \wedge \phi \mathrm{C}_1\Big)\Big)\Bigg) \left(\lambda e.\text{gacc } \mathbf{S}_{(14)b} \ e \ \phi\right) \\
&\to_\beta \lambda\phi.\exists\Big(\lambda s.\text{named "Smith"} s \wedge \exists\Big(\lambda n.\text{named "New York"} n \wedge \neg\mathbf{live}\, s \ n \ \wedge \\
&\qquad \neg\mathbf{seem}\ \left(\exists\lambda y.\mathbf{girlfriend}\, y \wedge \mathbf{has}\, s \ y\right) \wedge \left(\lambda e.\text{gacc } \mathbf{S}_{(14)b} \ e \ \phi\right) \mathrm{C}_1\Big)\Big) \\
&\to_\beta \lambda\phi.\exists\Big(\lambda s.\text{named "Smith"} s \wedge \exists\Big(\lambda n.\text{named "New York"} n \wedge \neg\mathbf{live}\, s \ n \ \wedge \\
&\qquad \neg\mathbf{seem}\ \left(\exists\lambda y.\mathbf{girlfriend}\, y \wedge \mathbf{has}\, s \ y\right) \wedge \text{gacc } \mathbf{S}_{(14)b} \ \mathrm{C}_1 \ \phi\Big)\Big)
\end{aligned}
$$

The computation continues in the following subterm, where the interpretation $\mathbf{S}_{(14)b}$ is applied to the context terms, the selection function calls are evaluated and the referents are returned:

$$
\begin{aligned}
&\mathbf{S}_{(14)b} \ \mathrm{C}_1 \ \phi \\
&\to_\beta^* \mathbf{visit}\ \left(\text{sel}(\lambda x.\text{named "Smith"} x)\mathrm{C}_1\right) \left(\text{sel}(\lambda x.\text{named "New York"} x)\mathrm{C}_1\right) \\
&\wedge \phi \left(\text{upd}_O \left(\mathbf{visit}\,(\text{sel}(\lambda x.\text{named "Smith"} x)\mathrm{C}_1)(\text{sel}(\lambda x.\text{named "New York"} x)\mathrm{C}_1), \mathrm{C}_1\right)\right) \\
&= \mathbf{visit}\, s \ n \wedge \phi \left(\text{upd}_O \left(\mathbf{visit}\, s \ n, \mathrm{C}_1\right)\right)
\end{aligned}
$$

Computing the context update from C_1 to C_2 to be passed to the continuation:

$$
\begin{aligned}
\mathrm{C}_2 &:= \text{upd}_O \left(\mathbf{visit}\, s \ n, \mathrm{C}_1\right) \\
&= \langle L_1, \Delta_1, \Gamma_1, \neg\mathbf{seem}\ \left(\exists\lambda y.\mathbf{girlfriend}\, y \wedge \mathbf{has}\, s \ y\right) \wedge \mathbf{visit}\, s \ n, \langle E_{\text{girl}}\rangle\rangle
\end{aligned}
$$

where according to Definition 3.15, E_{girl} is given by the following instances D_{girl} of Δ

and G_{girl} of Γ:

$$\begin{aligned}
E_{\text{girl}} =& D_{\text{girl}} \wedge G_{\text{girl}} \\
D_{\text{girl}} =& \Big(\exists(\lambda z.\textbf{girlfriend}\, z \wedge \textbf{has}\, s\ z \wedge \textbf{live-apart}\, s\ z\ n)\Big) \\
& \quad \rightarrow \neg\textbf{seem}\ \Big(\exists(\lambda z.\textbf{girlfriend}\, z \wedge \textbf{has}\, s\ z)\Big) \wedge \\
& \Big(\exists(\lambda z.\textbf{girlfriend}\, z \wedge \textbf{has}\, s\ z \wedge \textbf{live-apart}\, s\ z\ n)\Big) \rightarrow \textbf{visit}\, s\ n \\
G_{\text{girl}} =& \exists(\lambda z.\textbf{girlfriend}\, z \wedge \textbf{has}\, s\ z \wedge \textbf{live-apart}\, s\ z\ n)
\end{aligned}$$

The formula $L \wedge D_{\text{girl}} \wedge G_{\text{girl}}$ is consistent and

$$L \wedge D_{\text{girl}} \wedge G_{\text{girl}} \vdash \neg\textbf{seem}\ \big(\exists\lambda y.\textbf{girlfriend}\, y \wedge \textbf{has}\, s\ y\big) \wedge \textbf{visit}\, s\ n$$

Remark 3.28. Under the correspondence between conversational implicatures and explanations and predictions in the context logic, Grice's reading of B as hinting at Smith having a girlfriend in New York is captured by this analysis.

To capture the second interpretation of (14) discussed in Section 3.1.3, the context given in Example 3.25 and used in Example 3.26 is expanded as follows.

Example 3.29 (*Smith doesn't seem to have a girlfriend these days* discourse, second interpretation)**.** The second context to consider contains the defaults *being busy is a reason for (and usually means) not having a girlfriend, working away from home is a reason for (and usually means) regularly visiting somewhere* and *regularly visiting somewhere is a reason for (and usually means) being busy*. This is formalized with the following defaults and conjectures:

$$\begin{aligned}
\Delta' = \Delta \wedge \forall\Big(\lambda x.\textbf{busy}\, x \rightarrow \neg\exists(\lambda y.\textbf{girlfriend}\, y \wedge \textbf{has}\, x\ y)\Big) \wedge \\
\forall\Big(\lambda x.\Big(\lambda y.(\textbf{work}\, x\ y \wedge \neg\textbf{live}\, x\ y) \rightarrow \textbf{visit}\, x\ y\Big)\Big) \wedge \\
\forall\Big(\lambda x.\Big(\lambda y.\textbf{visit}\, x\ y \rightarrow \textbf{busy}\, x\Big)\Big) \\
\Gamma' = \Gamma \wedge \Big(\lambda x.\Big(\lambda y.\textbf{work}\, x\ y\Big)\Big)
\end{aligned}$$

With the addition of these defaults, the update from context C_0 to C_1 includes the following explanation as well:

$$\begin{aligned} E_{\text{work}} &= D_{\text{work}} \wedge G_{\text{work}} \\ D_{\text{work}} &= (\textbf{work}\, s\ n \wedge \neg \textbf{live}\, s\ n) \to \textbf{visit}\, s\ n \\ G_{\text{work}} &= \textbf{work}\, s\ n \end{aligned}$$

Then the maximal scenario of $(O_2 \wedge G_{\text{work}}, \Delta)$ includes the following conjunction, where $D_{\max}$ is the maximal instance of Δ:

$$\Big(\textbf{busy}\, s \to \neg\exists(\lambda y.\textbf{girlfriend}\, y \wedge \textbf{has}\, s\ y)\Big) \wedge \\ \textbf{visit}\, s\ n \wedge (\textbf{visit}\, s\ n \to \textbf{busy}\, s)$$

The maximal scenario generates the extension $Th(O_2 \wedge G_{\text{work}}, D_{\max})$, which includes the non-trivial propositions $\textbf{busy}\, s$ and $\neg\exists(\lambda y.\textbf{girlfriend}\, y \wedge \textbf{has}\, s\ y)$, and so it is predicted that Smith does not have a girlfriend because he is too busy.

3.3 Summary

This chapter made the following contributions:

- An analysis of Lebedeva [2012]'s approach to capturing conversational implicatures by proof-theoretic abduction. This showed that the simplified dynamization function does not preserve the incremental update property of dynamic conjunction, and so removes the ability to capture sentence-level phenomena such as the projection problem for presuppositions.

- A more complex context structure for $GL\chi$, motivated by the use of abduction in Lebedeva [2012]'s approach to conversational implicatures but extricated from the proof-theoretical representation. This is shown to provide a similar analysis to Lebedeva's approach, while also being readily implementable, preserving sentence-level properties of $GL\chi$ and being able to capture a wider range of conversational implicatures, i.e. those related to defaults, as in the second reading of (14) given in Section 3.1.3.

- Relatedly, an extension is suggested to Hobbs et al. [1993]'s hypothesis – adopted by Lebedeva [2012] – that abduced knowledge corresponds to the implicatures of an utterance. This is via the observation that with the new context structure, implicated content emerges as the result of both abduction in the form of explanations and in the form *predictions*.

The real utility of the approach will be seen in the next chapter, where it is applied to examples of conventional implicatures to analyse the relationship between these phenomena.

Chapter 4

Conventional Implicatures

The sibling of conversational implicatures "born into neglect" (Potts [2005a]), conventional implicatures were considered as far back as Frege [1892/1948] and brought to prominence when named by Grice [1975] for the sake of being set aside. Accounting for conventional implicatures as a class of meaning is complicated by the fact that the term is used to refer to a diverse body of lexical items, has at least two very distinct characterizations – Grice [1975] and Potts [2005b] – and is the subject of prominent claims of non-existence (Bach [1999]).

The conventional implicatures originally identified in Grice [1975] have been expanded to include adverbs *already, only, also, yet*; connectives *but, nevertheless, so, therefore*; implicative verbs *bother, manage, continue, fail*; subordinating conjunctions *although, despite, even though*; and "utterance modifiers" (Bach [1999]) *on the other hand, to get back to the point, in other words*. Their commonality is the contribution of meaning outside of what is explicitly said and, crucially, meaning attributed to a particular lexical item. With the former feature, they share the essence of conversational implicatures and with the latter, they look similar to presuppositions: therein lies the problem.

Formal treatments begin with Karttunen and Peters [1975]'s extension of Montague semantics. This associates with each phrase two expressions in intensional logic – the standard interpretation in Montague semantics and an interpretation of any contribution to implicatures of the utterance. This is partly achieved by *meaning postulates*, which encode certain other lexical items – and relations between them – into an interpretation. For example, the meaning postulate in the interpretation of *fail-to* is a formal expression associating '*x fails to P*' with '*x tried to P* or *y expected that x would P*'. In the case of *Mary failed to arrive*, the second disjunct – that someone expected Mary to arrive – obtains, and in the case of *my son failed to lift the piano*, the first disjunct – that he tried to lift the piano – obtains. In this way, the meaning of the lexical item producing the implicature is elaborated in a lexical semantic hard-coding.

As well as *fails*, Karttunen and Peters [1979] interpret *manage*, *too* and *even*. While these words are now commonly considered presupposition triggers, the work still plays a crucial role in formalizing conventional implicatures by influencing subsequent work and forming a historical link to presuppositions that means an insightful treatment of conventional implicatures needs to account for their differences. To this end, the treatment of a wider range of presuppositions in $GL\chi$ than presented in Chapter 2 will be addressed in Section 4.1.

Karttunen and Peters' description logic forms the basis of Potts [2005b]'s formalism, which itself is the cornerstone of contemporary conventional implicature semantics. Given a dearth of formal treatments – to which Potts attributes the debate around what is and is not a conventional implicature – Potts undertook a reformulation of conventional implicatures, based on Grice's remarks but divorced from the notion of implicature altogether – enforced by called them 'CIs':

> [CIs] are secondary entailments that cooperative speakers rarely use to express controversial propositions or carry the main themes of a discourse. Rather, CI expressions are used to guide the discourse in a particular direction or to help the hearer to better understand why the at-issue content is important at that stage. [Potts, 2005b, p.7]

CIs are defined as speaker-oriented commitments that are part of the conventional meaning of words and logically independent from at-issue content. They are evidenced not by the classical examples above but by supplemental expressions, such as appositives and non-restrictive relative clauses, and expressives, like 'damn'. As such, formal treatments of traditional conventional implicature items have not been given in Potts' CI logic. The CI logic is multidimensional, specifying types of at-issue content and not-at-issue content, as well as their interaction. In this way, and like Bach [1999]'s 'The Myth of Conventional Implicature', Potts rejects the doctrine that one sentence contains only one meaning, the result of which is the demotion in status of the meaning associated with conventional implicature.

While Potts' multidimensional logic for handling CIs spurred interest in formalizing this meaning class, the wave largely did not extend to Gricean conventional implicatures. One exception is SDRT, introduced in Chapter 3, which formalizes a range of both Pottsian and Gricean conventional implicatures. Hunter and Asher [2016] address CIs generated by appositives, while *but* and other discourse connectives and adverbials of contrast such as *however* have a treatment in SDRT using the discourse relation *contrast* (to be detailed in Section 4.2). It is similar to Karttunen and Peters' approach in that it elaborates the meaning of a singular lexical item, hard-coding certain relations to other lexical items.

Towards accounting for both Gricean conventional implicatures and Potts' CIs in $GL\chi$, we consider canonical lexical items from each class – *but* and supplements respectively. The idea is to see how far a common approach to conventional and conversational implicatures – using minimal, familiar tools – can proceed, and what insight into their relationship it can provide. Given the proximity of conventional implicatures to presuppositions, this chapter initially returns to the latter class to update their treatment with the new context structure and address presuppositions from sources other than referring expressions. *But* and supplements are then addressed in turn, including more details about their treatments in other formalisms, as well as examples using the new context structure in $GL\chi$.

4.1 Return to Presuppositions in $GL\chi$

In $GL\chi$, as in the original Montague semantics, most words are interpreted with atomic constants – there is no difference in the way verbs *love*, *hate* and *eat* are interpreted, for example:

$$\begin{aligned}\overline{[\![love]\!]} &= \lambda \mathbf{XY}.\mathbf{X}(\lambda \mathbf{x}.\mathbf{Y}(\lambda \mathbf{y}.\overline{\mathbf{love}}\,\mathbf{xy}))\\ \overline{[\![hate]\!]} &= \lambda \mathbf{XY}.\mathbf{X}(\lambda \mathbf{x}.\mathbf{Y}(\lambda \mathbf{y}.\overline{\mathbf{hate}}\,\mathbf{xy}))\\ \overline{[\![eat]\!]} &= \lambda \mathbf{XY}.\mathbf{X}(\lambda \mathbf{x}.\mathbf{Y}(\lambda \mathbf{y}.\overline{\mathbf{eat}}\,\mathbf{xy}))\end{aligned}$$

Their meaning is differentiated on the level of interpretation, in the regular logical sense of the word, such as in a model. Certain words, however, have more information encoded in their logical form. In $GL\chi$, these are the items that receive a non-standard translation from static to dynamic semantics, described by Shan [2005] as having *linguistic side-effects* – a term reflecting their treatment by analogy with computational side-effects.

A formalism needs principled reasons for which meaning to hard-code at the level of expression in a logical form and which meaning to locate in interpretation with respect to, say, a model. One reason for splitting an atom is where meaning associated with that atom exhibits different projection properties. For example, "Mary didn't quit smoking" negates the meaning that Mary quit smoking while retaining the meaning in"Mary quit smoking" that Mary smoked. The same reading does not apply to, say, "Mary loves smoking". This motivates splitting the interpretation of *quit* into the regular content and the presupposed content.

This notion of multiple dimensions of meaning is associated with both presuppositions and conventional implicatures, and so it warrants updating the treatment of presuppositions with the new context structure before considering conventional implicatures in $GL\chi$. Furthermore, Chapter 2 only presented the method of exception raising and handling for presuppositions associated with referring expression. Referring expressions sit squarely in the category of presuppositions, unlike other items such as *only, too, even* and implicative verbs like *failed* and *manage* that have been labelled both presupposition and conventional implicature.

4.1.1 Presuppositions from Factive and Aspectual Verbs in $GL\chi$

The other presupposition triggers that have been given treatments in $GL\chi$ are factive verbs (Lebedeva [2012]) and aspectual verbs (Grant [2017]). In the further work section of Lebedeva [2012], the approach used for referring expressions is shown to be applicable to the factive verb *know*. This is illustrated with the following sentence, which presupposes that John loves Mary:

(5) Tom knows that John loves Mary.

If the hearer does not know the content of the subordinate clause, it is accommodated by addition to the context; otherwise, only the main clause is added. This is captured in the interpetation of *know*, which has the following non-standard dynamization of the static interpretation from Montague semantics:

$$\overline{[\![knows]\!]} = \lambda \mathbf{Q}\mathbf{X}.\mathbf{X}(\lambda \mathbf{x}.\lambda e\phi.\mathbf{know}\,(\mathbf{x}e)(\mathbf{Q}e(\lambda e.\top)) \\ \wedge \phi \left(\mathrm{upd}\left(\mathbf{know}\,(\mathbf{x}e)(\mathbf{Q}e(\lambda e.\top)), e\right)\right)\Bigg)$$

$$\widetilde{[\![knows]\!]} = \lambda \mathbf{Q}\mathbf{X}.\mathbf{X}\Big(\lambda \mathbf{x}.\lambda e\phi.\mathbf{know}\,(\mathbf{x}e)(\mathrm{checkvalid}\ \mathbf{Q}e(\lambda e.\top)\ e) \\ \wedge \phi \left(\mathrm{upd}\left(\mathbf{know}\,(\mathbf{x}e)(\mathrm{checkvalid}\ \mathbf{Q}e(\lambda e.\top)\ e), e\right)\right)\Big)$$

The second argument, which takes the subordinate clause, is modified to include the function checkvalid, taking a proposition and a context as arguments and attempting to prove the proposition from the context:

$$\text{checkvalid } p\ e := \begin{cases} p & \text{if } \{e \vdash p\} \\ \text{raise } (\texttt{UnprovablePropExc}\ p) & \text{otherwise} \end{cases}$$

This is handled at discourse update with an exception handler similar to that given in Definition 3.2 for abducing propositions, except with regular context update rather than abduction:

$$\text{dupd } \mathbf{D}\ \mathbf{S} := \lambda\phi.\mathbf{D}(\lambda e.\text{gacc } \mathbf{S}\ e\ \phi) \tag{4.1a}$$

$$\text{gacc } \mathbf{S}\ e\ \phi := \text{checkprovable } \mathbf{S}\ e\ \phi \tag{4.1b}$$

$$\texttt{handle } (\texttt{AbsentIndividualExc}\ Q)\ \texttt{with}$$

$$\exists(\lambda x.(Qx) \wedge \text{gacc } \mathbf{S}\ (\text{upd}(Qx, e))\ \phi) \tag{4.1c}$$

$$\texttt{handle } (\texttt{UnprovablePropExc}\ F)\ \texttt{with}$$

$$\text{gacc } \mathbf{S}\ (\text{upd}(F, e))\ \phi \tag{4.1d}$$

In this way, the same projection behaviour as for referring expressions is achieved.

Based on this approach, Grant [2017] provides a treatment for aspectual verbs. The example verb is *quit*, as in the following sentence, which is said to presuppose that Mary used to smoke:

(6) Mary quit smoking.

This has a temporal notion, and so Grant introduces a modal operator **PAST**, as in **PAST**(**smoke** m), which captures "Mary smoked in the past". The non-standard dynamization of *quit* includes the function checkpastvalid, which tries to prove the proposition with the **PAST** operator applied to it:

$$\text{checkpastvalid } p\ e := \begin{cases} p & \text{if } e \vdash \mathbf{PAST}(p) \\ \text{raise } (\texttt{UnprovablePropExc}\ \mathbf{PAST}(p)) & \text{otherwise} \end{cases}$$

4.1.2 Updated Treatment with New Context Structure

These definitions can be tweaked for use with the new context structure by adjusting the condition used to define them from proof to set membership, as follows. For a conjunction of propositions $T = t_1 \wedge \ldots t_n$, let $T^* = \{t_1, \ldots, t_n\}$. Then for a context

$C = \langle L, \Delta, \Gamma, O, \mathcal{E} \rangle$,

$$\text{checkvalid } p \text{ C} := \begin{cases} p & \text{if } p \in L^* \cup \Delta^* \cup O^* \\ \text{raise } (\texttt{UnprovablePropExc } p) & \text{otherwise} \end{cases}$$

$$\text{checkpastvalid } p \text{ C}_n := \begin{cases} p & \text{if } p \in L^* \cup \Delta^* \cup O^* \\ \text{raise } (\texttt{UnprovablePropExc } \mathbf{PAST}(p)) & \text{otherwise} \end{cases}$$

The proof condition formally used is redundant with the context logic since it already contains logical consequences from the interaction of observations and lexical semantic information. The presupposed content can be found in the lexical semantic information and observation. This definition is an improvement based on the hypothesis that presupposed content should be explictly contained in the context, not merely provable from the context.

Finally, in the definition of discourse update the context update function upd needs to be replaced with the background update function upd_Δ, as in Definition 3.23. Equation (4.1d) becomes:

$$\text{gacc } \mathbf{S} \ (\text{upd}_\Delta(F, e)) \ \phi$$

Categorization as defeasible information corresponds to presupposed content having the property of being backgrounded.

After looking at *but* and supplements in $GL\chi$, comparison between implicatures and presuppositions can be made based on how they are formalized.

4.2 *But*

As a discourse connective, a natural comparison for *but* is *and*. Bach [1999] illustrates "the common wisdom" about the difference between the two with the following utterances:

(7) Shaq is huge but he is agile.

(8) Shaq is huge and he is agile.

Utterances (7) and (8) have the same truth conditions, the difference lying "not in what they say but merely in what is indicated by (the presence of) the word 'but"'. In this way, the meaning is considered elusively non-truth-conditional.

This "common wisdom" has its origins in Frege [1879]: "a speaker uses 'but' when he wants to hint that what follows is different from what might at first be supposed". Dummett [1973] counters that the contrast is "not necessarily one between what the second half of the sentence asserts, and what you would expect, knowing the first half to be true". Rather, it "is used to hint that there is some contrast, relevant to the context, between the two halves of the sentence: no more can be said, in general, about what kind of contrast is hinted at". Dummett illustrates with the following discourse, recently revived by Kripke [2017], involving a discussion about who to invite to give a talk:

(9) A: Robinson always draws large audiences.
B: He always draws large audiences, but he is in America for the year.

Firstly, utterance (9)a can be interpreted as conversationally implicating that Robinson should be invited, in order for it to be a relevant contribution to the conversation. Then the contrast in (9)b is between reasons for and against inviting Robinson, as opposed to between being in America and being a popular speaker. This makes for a more complex picture of *but* as not simply contrasting the clauses being joined.

Attempts to give a formal account for *but* follow a template for treatments of conventional implicatures, observed by Potts [2015], that associates independent dimensions of meaning with a word. In the case of *but*, this is the pair $(p \wedge q, R(p,q))$, where R represents a relation of contrast between p and q, or surprise at q given q. The relation $R(p,q)$ is treated as a presupposition of the utterance. Under such an interpretation, *but* has the same truth conditions as 'and', albeit with this extra dimension. This approach leaves the nature of the contrast unspecified and locates it between the conjuncts of *but*. A formalization that can determine the contrast, given that it need not be directly between the two clauses joined by *but*, makes an important contribution as it can capture more of the data.

A more complex treatment is undertaken in SDRT, interpreting *but* via the discourse relation $\mathit{Contrast}(\alpha, \beta)$, where α, β are elementary discourse units and K_α, K_β are their discourse representation structures – their sentence level interpretations. The semantics of *Contrast* specifies a partial isomorphism between their discourse representation structures, ensuring that their semantic structures are similar, that there is a *contrasting theme* between K_α and K_β, computed on the basis of the partial isomorphism. That is, their structures must be similar but their contents must be different, maximized when one argument is the negation of the other. It also encodes a property called the *Satisfaction Schema for Veridical Rhetorical Relations*. It is not easy to capture how this representation works without presenting the full details of SDRT, but we note that it is similar to the basic treatment in that it uses a non-specific contrast relation, albeit other relations may hold between α and β, and with other discourse units, so that a greater sense of the contrast is captured.

Returning to (7), there are other features of its meaning to account for. Bach [1999] considers "the most natural way of taking *but* especially out of context" is "as indicating that being huge tends to preclude being agile". However, it is not clear whether "out of context" means by the conventional meaning of the words alone, or instead refers to conventional meanings alongside knowledge of the world. Any contrast between agility and being huge comes from knowledge of the world – that people exist in restricted spaces surrounded by objects that make swift movement easier for smaller people, or that it is biologically the case that great size generally precludes agility. To clarify, consider the following variations:

(10) (a) Shaq is huge but he is rich.

(b) Shaq is huge but he is small.

Although muddied by our knowledge of the world, (7) is comparable to (10)a in suggesting a contrast between two logically independent properties, based on definition alone. On the other hand, (10)b has a conventional contrast, however it is infelicitous for the very reason that it is a conventional contradiction, not the product of defeasible reasoning. The challenge here is to capture these distinctions.

Furthermore, the same utterance with *but* can be associated with different contrasts. Another context for (7) is provided by Bach, in which there is a discussion of NBA centers, who are usually huge and agile. Someone says 'Shaq is huge and clumsy', to which the reply given is (7). In this case, it is necessary to capture that the use of *but* is related to the contradiction between the initial statement and the response.

In sum, debate about the nature of *but* centres on three features:

- the location of the contrast associated with *but*, given that it does not necessarily contrast the two clauses it connects;
- whether "more can be said, in general," about the nature of the contrast, accounting for the variance from surprise to apparent contradiction; and
- the status of the contrast as being merely "hinted at", compared to the regular content being connected.

We proceed by formalizing the above examples to make the observations precise and address these questions.

4.2.1 Formalization

To investigate the observed context-dependence of *but*, rather than hard-coding a notion of contrast in its logical form we assign it the same interpretation as *and*:

$$\overline{[\![but]\!]} = \lambda \mathbf{AB}.\lambda e\phi.\mathbf{A}e(\lambda e.\mathbf{B}e\phi)$$

Using the context logic, the contrastive meaning associated with *but* in the above examples can be identified in the reasoning in the context. We begin by formalizing discourse (9).

Example 4.1 (*Robinson always draws large audiences* discourse)**.** We want to represent the following context: *being popular is a reason for inviting someone, not being in Oxford is a reason for not being invited* and *if someone is in Oxford then they are not in America*. Paraphrase *draws large audiences* with *popular* and let $\mathrm{C} = \langle L, \Delta, \Gamma, \top, \top \rangle$ as follows:

$$\begin{aligned}
L =& \top \\
\Delta =& \forall(\lambda x.\text{named "Robinson"}x \to \mathbf{human}\,x) \wedge \\
& \forall(\lambda x.\text{named "Robinson"}x \to \mathbf{male}\,x) \wedge \\
& \forall(\lambda x.\mathbf{popular}\,x \to \mathbf{invite}\,x) \wedge \\
& \forall(\lambda x.\mathbf{invite}\,x \to \mathbf{in\text{-}oxford}\,x) \wedge \\
& \forall(\lambda x.\mathbf{in\text{-}oxford}\,x \to \neg\mathbf{in\text{-}america}\,x) \\
\Gamma =& \forall(\lambda x.\mathbf{popular}\,x) \wedge \forall(\lambda x.\mathbf{invite}\,x) \wedge \\
& \forall(\lambda x.\mathbf{in\text{-}oxford}\,x)
\end{aligned}$$

The sentence-level interpretation of $\mathbf{S}_{(9)a}$ is given by:

$$\begin{aligned}
\mathbf{S}_{(9)a} \to_{\beta}^{*} \lambda e\phi.&\mathbf{popular}\left(\text{sel}\left(\lambda x.(\text{named "Robinson"})x\right) e\right) \\
&\wedge \phi\left(\text{upd}_{O}\left(\mathbf{popular}\left(\text{sel}\left(\lambda x.(\text{named "Robinson"})x\right) e\right), e\right)\right)
\end{aligned}$$

This sentence is to be interpreted in the following discourse:

$$\mathbf{D}_0 = \lambda\phi.\exists\Big(\lambda r.\text{named "Robinson"}r \wedge \phi\left(\text{upd}_\Delta(\text{named "Robinson"}r, \mathrm{C})\right)\Big)$$

Updating C with this background information, the initial context is now:

$$\begin{aligned}\mathrm{C}_0 &= \text{upd}_\Delta(\text{named "Robinson"}r, \mathrm{C})\\ &= \langle L, \Delta \wedge \text{named "Robinson"}r, \Gamma, \top, \top\rangle\end{aligned}$$

Discourse $\mathbf{D}_0$ is updated with utterance $(9)a$ as follows:

$$\begin{aligned}\mathbf{D}_1 &:= \text{dupd}\ \mathbf{D}_0\ \mathbf{S}_{(9)a}\\ &= \lambda\phi.\mathbf{D}_0(\lambda e.\text{gacc}\ \mathbf{S}_{(9)a}\ e\ \phi)\\ &= \lambda\phi.\Big(\lambda\phi.\exists\Big(\lambda r.\text{named "Robinson"}r \wedge \phi\mathrm{C}_0\Big)\Big)(\lambda e.\text{gacc}\ \mathbf{S}_{(9)a}\ e\ \phi)\\ &\to_\beta \lambda\phi.\exists\Big(\lambda r.\text{named "Robinson"}r \wedge (\lambda e.\text{gacc}\ \mathbf{S}_{(9)a}\ e\ \phi)\mathrm{C}_0\Big)\\ &\to_\beta \lambda\phi.\exists\Big(\lambda r.\text{named "Robinson"}r \wedge \text{gacc}\ \mathbf{S}_{(9)a}\ \mathrm{C}_0\ \phi\Big)\end{aligned}$$

Computation proceeds in the subterm $\mathbf{S}_{(9)a}\ \mathrm{C}_0\ \phi$ with evaluation of the sel function calls:

$$\begin{aligned}&\mathbf{S}_{(9)a}\ \mathrm{C}_0\ \phi\\ &= \lambda e\phi.\mathbf{popular}\ \Big(\text{sel}\ (\lambda x.(\text{named "Robinson})x)\ e\Big)\\ &\qquad \wedge \phi\left(\text{upd}_O\left(\mathbf{popular}\ \Big(\text{sel}\ (\lambda x.(\text{named "Robinson})x)\ e\Big), e\right)\right)\mathrm{C}_0\ \phi\\ &\to_\beta^* \mathbf{popular}\ \Big(\text{sel}\ (\lambda x.(\text{named "Robinson})x)\ \mathrm{C}_0\Big)\\ &\qquad \wedge \phi\left(\text{upd}_O\left(\mathbf{popular}\ \Big(\text{sel}\ (\lambda x.(\text{named "Robinson})x)\ \mathrm{C}_0\Big), \mathrm{C}_0\right)\right)\\ &= \mathbf{popular}\, r \wedge \phi\Big(\text{upd}_O\left(\mathbf{popular}\, r, \mathrm{C}_0\right)\Big)\end{aligned}$$

Looking at the context update:

$$\begin{aligned}\mathrm{C}_1 &= \text{upd}_O(\mathbf{popular}\, r, \mathrm{C}_0)\\ &= \langle L_0, \Delta_0, \Gamma_0, \mathbf{popular}\, r, \top\rangle\end{aligned}$$

The observation has no explanations but consider the extensions of the context. A maximal instance of Δ_1 is given by $D_{\max}$:

$$D_{\max} = (\text{named “Robinson”}r \rightarrow \mathbf{human}\, r) \wedge (\text{named “Robinson”}r \rightarrow \mathbf{male}\, r) \wedge$$
$$(\mathbf{popular}\, r \rightarrow \mathbf{invite}\, r) \wedge (\mathbf{invite}\, r \rightarrow \mathbf{in\text{-}oxford}\, r) \wedge (\mathbf{in\text{-}oxford}\, r \rightarrow \neg\mathbf{in\text{-}america}\, r)$$
$$\wedge \text{ named “Robinson”}r$$

This is the only maximal instance of Δ_1 since the domain of C_1 is $\{r\}$. This means there is only one extension of C_1, generated by the following maximal scenario of (O_1, Δ_1):

$$O_1 \wedge D_{\max} = \mathbf{popular}\, r \wedge D_{\max}$$

The extension, $Th(O_1 \wedge D_{\max})$, includes $\mathbf{invite}\, r$, $\mathbf{in\text{-}oxford}\, r$ and $\neg\mathbf{in\text{-}america}\, r$. Thus the theory of context predicts that Robinson should be invited, and that for this suggestion to be conversationally cooperative, Robinson is in Oxford and so not in America. In this way, the predictions correspond to possible conversational implicatures of utterance (9)a.

Returning to the main term of the discourse interpretation, there are no exceptions to be handled so the updated discourse is:

$$\mathbf{D}_1 = \lambda\phi.\exists\Big(\lambda r.\text{named “Robinson”}r \wedge \mathbf{popular}\, r \wedge \phi \mathrm{C}_1\Big)$$

Proceeding with the computation of the discourse, the sentence-level interpretation of B’s utterance is given by:

$$\mathbf{S}_{(9)b} \rightarrow^*_\beta \lambda e\phi.\mathbf{popular}\ \left(\mathrm{sel}\left(\lambda x.\mathbf{male}\, x \wedge \mathbf{human}\, x\right) e\right)$$
$$\wedge\ \mathbf{in\text{-}america}\ \Big(\mathrm{sel}\left(\lambda x.\mathbf{male}\, x \wedge \mathbf{human}\, x\right) e_p\Big)$$
$$\wedge\ \phi \left(\mathrm{upd}_O \left(\mathbf{in\text{-}america}\ \Big(\mathrm{sel}\left(\lambda x.\mathbf{male}\, x \wedge \mathbf{human}\, x\right) e_p\Big), e_p\right)\right)$$

where $e_p = \mathrm{upd}_O \Big(\mathbf{popular}\ \left(\mathrm{sel}\left(\lambda x.\mathbf{male}\, x \wedge \mathbf{human}\, x\right) e\right), e\Big)$. The discourse updated with this utterance is given by $\mathbf{D}_2$:

$$\begin{aligned}\mathbf{D}_2 &= \mathrm{dupd}\ \mathbf{D}_1\ \mathbf{S}_{(9)b} \\ &= \lambda\phi.\mathbf{D}_1(\lambda e.\mathrm{gacc}\ \mathbf{S}_{(9)b}\ e\ \phi) \\ &= \lambda\phi.\Big(\lambda\phi.\exists\Big(\lambda r.\mathrm{named}\ \text{“Robinson”}r \wedge \mathbf{popular}\, r \wedge \phi \mathrm{C}_1\Big)\Big)(\lambda e.\mathrm{gacc}\ \mathbf{S}_{(9)b}\ e\ \phi) \\ &\to_\beta \lambda\phi.\exists\Big(\lambda r.\mathrm{named}\ \text{“Robinson”}r \wedge \mathbf{popular}\, r \wedge (\lambda e.\mathrm{gacc}\ \mathbf{S}_{(9)b}\ e\ \phi)\mathrm{C}_1\Big) \\ &\to_\beta \lambda\phi.\exists\Big(\lambda r.\mathrm{named}\ \text{“Robinson”}r \wedge \mathbf{popular}\, r \wedge \mathrm{gacc}\ \mathbf{S}_{(9)b}\ \mathrm{C}_1\ \phi\Big)\end{aligned}$$

The computation proceeds in the subterm $\mathbf{S}_{(9)b}\ \mathrm{C}_1\ \phi$ with evaluation of the sel function calls to determine the pronoun referents:

$$\begin{aligned}\mathbf{S}_{(9)b}\mathrm{C}_1\phi \to^*_\beta\ & \mathbf{popular}\ \big(\mathrm{sel}\,(\lambda x.\mathbf{male}\, x \wedge \mathbf{human}\, x)\,\mathrm{C}_1\big) \\ & \wedge \mathbf{in\text{-}america}\ \Big(\mathrm{sel}\,(\lambda x.\mathbf{male}\, x \wedge \mathbf{human}\, x)\,\mathrm{C}_p\Big) \\ & \wedge \phi\left(\mathrm{upd}_O\left(\mathbf{in\text{-}america}\ \Big(\mathrm{sel}\,(\lambda x.\mathbf{male}\, x \wedge \mathbf{human}\, x)\,\mathrm{C}_p\Big), \mathrm{C}_p\right)\right) \\ =\ & \mathbf{popular}\, r \wedge \mathbf{in\text{-}america}\, r \wedge \phi\left(\mathrm{upd}_O\left(\mathbf{in\text{-}america}\, r, \mathrm{upd}_O\left(\mathbf{popular}\, r, \mathrm{C}_1\right)\right)\right)\end{aligned}$$

where $\mathrm{C}_p = \mathrm{upd}_O\Big(\mathbf{popular}\ \big(\mathrm{sel}\,(\lambda x.\mathbf{male}\, x \wedge \mathbf{human}\, x)\,\mathrm{C}_1\big), \mathrm{C}_1\Big)$. The subterm passing the updated context to the continuation of the discourse is:

$$\phi(\mathrm{upd}_O(\mathbf{in\text{-}america}\, r, \mathrm{upd}_O(\mathbf{popular}\, r, \mathrm{C}_1)))$$

Let C_2 be the inner most context update, then:

$$\begin{aligned}\mathrm{C}_2 &= \mathrm{upd}_O(\mathbf{popular}\, r, \mathrm{C}_1) \\ &= \langle L_1, \Delta_1, \Gamma_1, \mathbf{popular}\, r, \top\rangle\end{aligned}$$

There is no explanation for the observation and the same predictions are made in the theory of context as before. [1] Performing the outer context update:

$$\begin{aligned}\mathrm{C}_3 &= \mathrm{upd}_O(\mathbf{in\text{-}america}\, r, \mathrm{C}_2) \\ &= \langle L_2, \Delta_2, \Gamma_2, \mathbf{popular}\, r \wedge \mathbf{in\text{-}america}\, r, \top\rangle\end{aligned}$$

[1] It is beyond the scope of this thesis to account for the meaning of reinforcing a statement, as in the repetition of “Robinson always draws large audiences”.

Again, there is no explanation for the observations; the predictions, however, are different. The maximal instance of Δ_3 is the same as that for Δ_1 and Δ_2, but there is no maximal scenario of (O_3, Δ_3) because the only candidate maximal scenario $O_3 \wedge D_{\max}$ is inconsistent, entailing both $\textbf{in-america}\, r$ and $\neg\textbf{in-america}\, r$:

$$O_3 \wedge D_{\max} = \textbf{popular}\, r \wedge \textbf{in-america}\, r \wedge (\textbf{popular}\, r \to \textbf{invite}\, r) \wedge \\ (\textbf{invite}\, r \to \textbf{in-oxford}\, r) \wedge (\textbf{in-oxford}\, r \to \neg\textbf{in-america}\, r)$$

The new theory of context does not include the prediction that Robinson should be invited.

In this example, the meaning associated with *but* can be located in the context change from C_2 to C_3. The content introduced by *but* has the effect of cancelling the implicatures of the preceding content. Modelling the context this way, there is no need to hard-code a contrast in the interpretation of *but* because the contrast arises automatically, as a result of reasoning in the context when the content is added. Unlike in the cases of presupposition in Section 4.1, there is no motivation to hard-code the additional dimension of meaning in *but*. This suggests viewing *but* as a pragmatic choice of connective to licence an inconsistency from one context to the next, corresponding to a *procedural* account of meaning whereby connectives guide the inferences made by the hearer of an utterance. This will be discussed further after formalizing (7) and variations (10)a and (10)b.

Example 4.2 (*Shaq is huge but he is agile*, first interpretation)**.** We want to represent the following context: *an entity called Shaq is usually a human, an entity called Shaq is usually male, being huge is a reason for (and usually means) not being agile, someone can be huge* and *being huge means not being small*. This can be formalized as $C = \langle L, \Delta, \Gamma, \top, \top \rangle$ as follows:

$$\begin{aligned} L =& \forall(\lambda x.\textbf{huge}\, x \to \neg\textbf{small}\, x) \wedge \\ & \forall(\lambda x.\neg\textbf{small}\, x \to \textbf{huge}\, x) \\ \Delta =& \forall(\lambda x.\text{named “Shaq”} x \to \textbf{human}\, x) \wedge \\ & \forall(\lambda x.\text{named “Shaq”} x \to \textbf{male}\, x) \wedge \\ & \forall(\lambda x.\textbf{huge}\, x \to \neg\textbf{agile}\, x) \\ \Gamma =& \forall(\lambda x.\textbf{huge}\, x) \end{aligned}$$

The sentence-level interpretation is:

$$\mathbf{S}_{(7)} \to_\beta^* \lambda e\phi.\mathbf{huge}\ \big(\mathrm{sel}(\lambda x.(\text{named “Shaq”})x)e\big) \wedge \mathbf{agile}\ \big(\mathrm{sel}(\lambda x.\mathbf{male}\, x \wedge \mathbf{human}\, x)e_h\big)$$
$$\wedge\, \phi \left(\mathrm{upd}_O \left(\mathbf{agile}\ \big(\mathrm{sel}(\lambda x.\mathbf{male}\, x \wedge \mathbf{human}\, x)e_h\big)\, , e_h\right)\right)$$

where $e_h = \mathrm{upd}_O \left(\mathbf{huge}\ \big(\mathrm{sel}(\lambda x.(\text{named “Shaq”})x)e\big)\, , e\right)$. The sentence is to be interpreted in the following discourse:

$$\mathbf{D}_0 = \lambda\phi.\exists\Big(\lambda s.\text{named “Shaq”} s \wedge \phi\Big(\mathrm{upd}_\Delta(\text{named “Shaq”}s, \mathrm{C})\Big)\Big)$$

With the update of background information, the initial context is now:

$$\mathrm{C}_0 = \langle L, \Delta \wedge \text{named “Shaq”}s, \Gamma, \top, \top\rangle$$

The update of $\mathbf{D}_0$ with $\mathbf{S}_{(7)}$ is computed as follows:

$$\begin{aligned}
\mathbf{D}_1 &= \mathrm{dupd}\ \mathbf{D}_0\ \mathbf{S}_{(7)} \\
&= \lambda\phi.\mathbf{D}_0(\lambda e.\mathrm{gacc}\ \mathbf{S}_{(7)}\ e\,\phi) \\
&= \lambda\phi.\Big(\lambda\phi.\exists\Big(\lambda s.\text{named “Shaq”}s \wedge \phi \mathrm{C}_0\Big)\Big)(\lambda e.\mathrm{gacc}\ \mathbf{S}_{(7)}\ e\,\phi) \\
&\to_\beta \lambda\phi.\exists\Big(\lambda s.\text{named “Shaq”}s \wedge (\lambda e.\mathrm{gacc}\ \mathbf{S}_{(7)}\ e\,\phi)\mathrm{C}_0\Big) \\
&\to_\beta \lambda\phi.\exists\Big(\lambda s.\text{named “Shaq”}s \wedge \mathrm{gacc}\ \mathbf{S}_{(7)}\ \mathrm{C}_0\,\phi\Big)
\end{aligned}$$

The computation proceeds in the subterm $\mathbf{S}_{(7)}\ \mathrm{C}_0\ \phi$ with evaluation of the sel function calls:

$$\mathbf{S}_{(7)}\ \mathrm{C}_0\ \phi \to_\beta^* \mathbf{huge}\ \big(\mathrm{sel}(\lambda x.(\text{named “Shaq”})x)\mathrm{C}_0\big) \wedge \mathbf{agile}\ \big(\mathrm{sel}(\lambda x.\mathbf{male}\, x \wedge \mathbf{human}\, x)\mathrm{C}_h\big)$$
$$\wedge\, \phi \left(\mathrm{upd}_O \left(\mathbf{agile}\ \big(\mathrm{sel}(\lambda x.\mathbf{male}\, x \wedge \mathbf{human}\, x)\mathrm{C}_h\big)\, , \mathrm{C}_h\right)\right)$$

where

$$\mathrm{C}_h = \mathrm{upd}_O \left(\mathbf{huge}\ \big(\mathrm{sel}(\lambda x.(\text{named “Shaq”})x)\mathrm{C}_0\big)\, , \mathrm{C}_0\right)$$

Inside $\mathrm{sel}(\lambda x.(\text{named "Shaq"})x)\mathrm{C}_0$, a proof of the following can be found:

$$\mathrm{C}_0 \vdash (\lambda x.(\text{named "Shaq"})x)s$$

so the selection function retrieves the individual s. Since $\Delta_0 \vdash$ named "Shaq"$s \rightarrow$ **human** s and $\Delta_0 \vdash$ named "Shaq"$s \rightarrow$ **male** s, a proof of the following can be found:

$$\mathrm{C}_h \vdash (\lambda x.\mathbf{male}\, x \wedge \mathbf{human}\, x)s$$

so the second selection function also retrieves s:

$$\mathbf{S}_{(7)}\ \mathrm{C}_0\ \phi \rightarrow_{eval} \mathbf{huge}\, s \wedge \mathbf{agile}\, s \wedge \phi \left(\mathrm{upd}_O \left(\mathbf{agile}\, s, \mathrm{upd}_O \left(\mathbf{huge}\, s, \mathrm{C}_0 \right) \right) \right)$$

Returning to the entire discourse term, we then have:

$$\mathbf{D}_1 = \lambda\phi.\exists \left(\lambda s.\text{named "Shaq"}s \wedge \mathbf{huge}\, s \wedge \mathbf{agile}\, s \wedge \phi \left(\mathrm{upd}_O \left(\mathbf{agile}\, s, \mathrm{upd}_O (\mathbf{huge}\, s, \mathrm{C}_0) \right) \right) \right)$$

Beginning with the innermost context update:

$$\begin{aligned} \mathrm{C}_1 &= \mathrm{upd}_O(\mathbf{huge}\, s, \mathrm{C}_0) \\ &= \langle L_0, \Delta_0, \Gamma_0, \mathbf{huge}\, s, \top \rangle \end{aligned}$$

There are no explanations. To determine any predictions, consider extensions of (O_1, Δ_1). A maximal instance of Δ_1 is given by $D_{\max}$:

$$\begin{aligned} D_{\max} =& (\text{named "Shaq"}s \rightarrow \mathbf{human}\, s) \wedge \\ & (\text{named "Shaq"}s \rightarrow \mathbf{male}\, s) \wedge \\ & (\mathbf{huge}\, s \rightarrow \neg\mathbf{agile}\, s) \wedge \\ & \text{named "Shaq"}s \end{aligned}$$

Since the domain of C_1 is $\{s\}$, $D_{\max}$ is the only maximal instance of Δ_1. Therefore, there is only one extension and it is generated by the following maximal scenario:

$$\begin{aligned} O_1 \wedge D_{\max} = \mathbf{huge}\, s \wedge (\text{named "Shaq"}s \rightarrow \mathbf{human}\, s) \wedge \\ (\text{named "Shaq"}s \rightarrow \mathbf{male}\, s) \wedge (\mathbf{huge}\, s \rightarrow \neg\mathbf{agile}\, s) \wedge \text{named "Shaq"}s \end{aligned}$$

Since $\neg\mathbf{agile}\, s$ is in $Th(O_1 \wedge D_{\max})$, it is predicted by the theory of context.

Performing the second context update:

$$\begin{aligned} C_2 &= \mathrm{upd}_O(\mathbf{agile}\, s, C_1) \\ &= (L_1, \Delta_1, \Gamma_1, \mathbf{huge}\, s \wedge \mathbf{agile}\, s, \top) \end{aligned}$$

This time, there is no extension because the only candidate, $Th(O_2 \cup D_{\max})$ is inconsistent, containing both $\neg\mathbf{agile}\, s$ and $\mathbf{agile}\, s$.

The formalization suggests that *Shaq is huge* can be interpreted as conversationally implicating that Shaq is not agile. The second clause of *but* is then viewed as cancelling the implicature of the first clause. As in the previous example, the role of *but* is to introduce content that requires an implicature associated with the immediately preceding content to be abandoned.

We now show how this formalization also accounts for infelicitous usages of *but*, in particular utterances (10)b and (10)a.

Example 4.3 (# *Shaq is huge but he is small*)**.** Utterance (10)b has an analogous sentence-level interpretation to (7), found by replacing the constant **agile** with the constant **small**:

$$\begin{aligned} \mathbf{S}_{(7)} \to^*_\beta\, & \lambda e\phi.\mathbf{huge}\ \big(\mathrm{sel}(\lambda x.(\text{named “Shaq”})x)e\big) \wedge \mathbf{small}\ \big(\mathrm{sel}(\lambda x.\mathbf{male}\, x \wedge \mathbf{human}\, x)e_h\big) \\ & \wedge \phi\left(\mathrm{upd}_O\left(\mathbf{small}\ \big(\mathrm{sel}(\lambda x.\mathbf{male}\, x \wedge \mathbf{human}\, x)e_h\big), e_h\right)\right) \end{aligned}$$

However, (10)b is not a felicitous utterance. This is not captured on the level of sentence, but it can be located on the level of discourse. Interpreting with respect to the same discourse as Example (4.2) and resolving the pronoun referents gives the following interpretation:

$$\lambda\phi.\mathbf{huge}\, s \wedge \mathbf{small}\, s \wedge \phi\left(\mathrm{upd}_O\left(\mathbf{small}\, s, \mathrm{upd}_O\left(\mathbf{huge}\, s, C_0\right)\right)\right)$$

The first update $C_1 = \mathrm{upd}_O\left(\mathbf{huge}\, s, C_0\right)$ is the same as before; the update from C_1 to C_2 is as follows:

$$\begin{aligned} C_2 &:= \mathrm{upd}_O(\mathbf{small}\, s, C_1) \\ &= \langle L_1, \Delta_1, \Gamma_1, \mathbf{huge}\, s \wedge \mathbf{small}\, s, \top\rangle \end{aligned}$$

In this case, there is an inconsistency in context C_2 itself, since

$$L_2 \wedge O_2 \vdash \neg \mathbf{small}\, s \wedge \mathbf{small}\, s$$

Rather than having a contradiction between contexts, the contradiction is within a context, which is not permitted by the logic. Thus we propose the following condition in $GL\chi$ for an utterance to be infelicitous.

Definition 4.4 (Infelicity condition)**.** Utterance **S** in context C_n is infelicitous if the context updated with **S**, C_{n+1}, is inconsistent.

This example is particularly important because it reveals a flaw in prevailing accounts of *but* – both the classical story and more recent discourse-based analyses. In SDRT, for example, the contrast relation has a notion of being maximised when one argument is the negation of the other. However, this example shows that such usage can be infelicitous, and demonstrates how the context logic in $GL\chi$ can capture this.

The final example is another infelicitous utterance containing *but*, for which the framework is also able to account.

Example 4.5 (# *Shaq is huge but he is rich*)**.** The sentence level interpretation of (10)a is again analoguous to that of (7), with the constant **agile** replaced by the constant **rich**. Therefore, interpretation in an empty discourse in context C_0 is given by:

$$\lambda\phi.\mathbf{huge}\, s \wedge \mathbf{rich}\, s \wedge \phi\left(\mathrm{upd}_O\left(\mathbf{rich}\, s, \mathrm{upd}_O\left(\mathbf{huge}\, s, C_0\right)\right)\right)$$

The first update is as before:

$$\begin{aligned} C_1 &= \mathrm{upd}_O(\mathbf{huge}\, s, C_0) \\ &= \langle L_0, \Delta_0, \Gamma_0, \mathbf{huge}\, s, \top\rangle \end{aligned}$$

The update from C_1 to C_2 is now:

$$\begin{aligned} C_2 &= \mathrm{upd}_O(\mathbf{rich}\, s, C_1) \\ &= \langle L_1, \Delta_1, \Gamma_1, \mathbf{huge}\, s \wedge \mathbf{rich}\, s, \top\rangle \end{aligned}$$

Context C_2 has the same predictions as context C_2 in Example 4.2, meaning there is no contradiction between the theory of context from C_1 to C_2.

In this case, the utterance is infelicitous because the condition for *but* being the pragmatic choice of connective is not satisfied. Based on this instance, another sufficient condition for utterance infelicity is given by first proposing a characterization of *but*.

Proposal 4.6 (Characterization of *but* in $GL\chi$)**.** *Suppose* but *conjoins clauses a and b, interpreted as propositions* $[\![a]\!]$ *and* $[\![b]\!]$*, and* $[\![a]\!]$ *is interpreted in context* C_{n-1}*. Then there is a proposition* $p \in I(\mathrm{C}_n)$ *and a proposition q enatiled by* C_{n+1} *such that* $p \wedge q \vdash \bot$*. That is, there is a defeasible contradiction between contexts.*

Definition 4.7 (Infelicity condition)**.** Utterance **S** in context C_n is infelicitous if the conditions of pragmatic usage on the lexical items in **S** contains are not met by its interpretation in context C_n.

Conditions of pragmatic usage refer to conditions like those given in Proposal 4.6 for *but*, and the definition anticipates that analysis of other conventional implicature-related items will motivate specifying conditions of pragmatic usage for these items.

Returning to (10)a, suppose it is uttered in a conversation between Shaq's friends, who are considering who to invite on an expensive caving holiday to a remote island. Speaker B suggests inviting Shaq, with the discourse continuing:

A. Shaq is huge! He's far too big to go caving.

B. Shaq is huge but he is rich.

Utterance (10)a is felicitous in this context, which could be given as *inviting Shaq is a possibility, caving in a remote area is expensive, being rich is a reason for inviting someone on an expensive trip, being huge tends to make caving difficult* and *being unable to go caving is a reason for not inviting someone on a caving trip.* In this context, when *he is rich* is added, there will be an inconsistency between the new context and the previous one, between a reason to invite Shaq and a reason against inviting Shaq. This illustrates the context-dependence of *but*, and how $GL\chi$ with reasoning in the context can account for it.

4.2.2 Discussion

Applying context logic analyses in $GL\chi$ to Examples (9), (7), (10)b and (10)a motivates a theory of *but* as the pragmatic choice of connective for introducing content into a context with which it is 'defeasibly inconsistent'. Supposing explanations and predictions correspond to implicatures, this could also be framed as *but* introducing content that 'cancels' an implicature of the context in which it is uttered. It remains to consider more data to test these hypotheses, but it forms an interesting link between conversational and (alleged) conventional implicature items.

This conception of *but* has similarities with a procedural theory of meaning (Blakemore [1987]). This approach accounts for linguistic puzzles by analysing certain lexical items as encoding procedures rather than mapping to concepts. Escandell-Vidal et al. [2011] explain:

> The major claim in Blakemore's book is that... linguistic meaning is not confined to determining truth-conditions, but it also plays a role in some non-truth-conditional aspects of utterance interpretation... Discourse connectives can feed the system of inferential rules: some of them introduce premises (after all, moreover) and conclusions (therefore), which are used to strengthen contextual assumptions; others, such as so, can point to implications; finally, other particles encode instructions for the hearer to abandon existing assumptions, as is the case with connectives of denial and contrast, such as but, however and nevertheless (Blakemore 1987, 1988, 1992).

The treatment of *but* developed has not been guided by this theory, instead being motivated by getting as much as possible with as little machinery as possible. It could, however, been seen as consistent with a procedural theory of meaning by identifying that the effect of *but* is to abandon certain defeasible information. Furthermore, while developing a pragmatic theory of *but* deviates from standard "conventional" – in the sense of lexically-encoded – treatments, there is precedent: the standard treatment of presuppositions is as a conventional feature of language, however there are important non-conventional, pragmatic treatments, such as Schlenker [2008].

Given that the contrast associated with *but* can be calculated from interaction with the context, hard-coding the contrast in the interpretation of *but* could be used to automatically generate context. This is an important observation for two reasons: it suggests an approach to the problem of automatically generating utterance context, but it also reveals how a computational motivation for a formal semantics can be at odds with linguistic enquiry. Unconsciously preferring the computationally advantageous approach and using it to draw conclusions about the nature of *but* would corroborate the conventional theory of *but* as lexically encoding a contrast. In this way, our two motivations for developing a semantics of natural language – understanding linguistic phenomena and developing tools that could be used in natural language processing – diverge here.

4.3 Supplements

Supplements, alongside expressives, are used as evidence for Potts' reformulation of Grice's definition of conventional implicatures as CIs. Supplemental expressions take different forms, as in the following examples from Potts [2005b]:

(11) Ames was, as the press reported, a successful spy.

(12) Ames, who stole from the FBI, is now behind bars.

(13) Ames, the former spy, is now behind bars.

The underlined constructions are examples of *as-parentheticals, supplementary relatives* and *nominal appositives* respectively.

The formal framework has two dimensions to capture the independence of CI and at-issue content.[2] Potts introduces this distinction by defining different types for at-issue content and CI content – at-issue entities e^a, truth values t^a and worlds s^a as well as CI entities e^c, truth values t^c and worlds s^c. These are explained:

> In essence, the types regulate semantic composition in the same way that natural language syntactic categories regulate the projection of category labels in syntactic structures.([Potts, 2005b, p.55]

From these types, there are two rules governing how function types are formed.

Definition 4.8 (Typing rules in Potts' multidimensional logic of CIs)**.** If σ, τ are at-issue types, then $\sigma \to \tau$ is an at-issue type. If σ is an at-issue type and τ is a CI type, then $\sigma \to \tau$ is a CI type.[3]

The second rule is how the two kinds of meaning interact: note that for σ an at-issue type and τ a CI type, then $\tau \to \sigma$ is not a well-formed type.

Potts' analysis of supplements centres on interpretations of the comma, of which there are "a handful", characterized by taking at-issue content to CI content:

$$\textsc{comma} \rightsquigarrow \lambda f \lambda x. f(x) : ((e^a \to t^a) \to (e^a \to t^c))$$

[2]Later work stemming from this has remarked that this independence is too strict, observing interaction between the content. See, for example, Bekki and McCready [2015], AnderBois et al. [2010].

[3]Potts uses $\langle \sigma, \tau \rangle$ notation for this; we replace this to make the notation closer to ours.

From the interaction stipulated by the typing rule in Definition 4.8, Potts demonstrates – and so accounts for – the properties of CI meaning as nondeniable, antibackgrounded, non-restrictive and scopeless.

Asher [2000] provides a treatment of parentheticals in SDRT, and Hunter and Asher [2016] uses SDRT to account for at-issue and not-at-issue meaning, using discourse structure. These look at the relationships between elementary discourse units, rather than interpretations of individual lexical items, and so is different from Potts' approach and the one we will take. Appositives attach to a main clause, and whether an appositive is accessible depends on how it is attached to the main clause.

To illustrate what is involved in capturing supplements, consider the following example from Potts [2005b]:

(14) Ed's claim, which is based on extensive research, is highly controversial.

Potts takes this utterance to mean "my primary intention is to arrive at an information state that entails the truth of the proposition that Ed's claim is highly controversial". Potts explains the meaning contributed by the supplement by comparing (14) with the utterance with the supplement removed:

(15) Ed's claim is highly controversial.

This constitutes the at-issue content of (14), with the supplementary relative CI content steering how the at-issue meaning should be taken. The effect of the CI content depends on the *context* of the utterance:

> With the CI content expressed by the supplementary relative, I provide a clue as to how the information should be received. This example is felicitous in a situation in which, for example, I want to convey to my audience that the controversy should not necessarily scare us away from Ed's proposal – after all, it is extensively researched. Or I might use the example with a group of detractors from Ed's claim. Then the supplementary relative could indicate that we cannot expect to dispel Ed's claim solely on the basis of its controversial nature. Potts [2005b]

The challenge here is to explain the proximity of Potts' description of CIs in general, and this example in particular, to Grice's notion of implicature as meaning outside of what is explicitly said. The goal is to formalize the intuition that CIs provide "a clue as to how the information should be received" and their relationship to implicatures.

More generally, Potts [2015] notes that "with the exception of appositives, the alleged conventional implicature content [of alleged conventional implicature items] is extremely hard to articulate". A formalism that can explain why this difficulty exists is desirable.

4.3.1 Formalization

Continuing by seeing how far we can get with a minimal approach, we do not ascribe a particular meaning to the supplement construction, viewing it as controlling the order of meaning.[4] The interpretation of the supplement construction in (14) is as a coordinating device:

$$\overline{[\![, \textit{which}\ldots,]\!]} = \lambda \mathbf{xPQ}.(\mathbf{Px} \mathbin{\overline{\wedge}} \mathbf{Qx}) \tag{4.2}$$

The order of update captures the order in which the contents is processed, and captures how the most recent clause is the one most open to being referenced in the continuation of the discourse. Formalizing (15) followed by (14) shows how the extra meaning can be viewed as arising from the interaction in the context: the meaning need not be encoded in a lexical item.

Example 4.9 (*Ed's claim is highly controversial*)**.** The interpretation of (15) is composed of the following interpretations:[5]

$$\begin{aligned}
\overline{[\![\textit{is highly controversial}]\!]} &= \lambda \mathbf{X}.\mathbf{X}(\lambda \mathbf{x}.\overline{\mathbf{controversial}}\,\mathbf{x}) \\
&= \lambda \mathbf{X}.\mathbf{X}\Big(\lambda \mathbf{x}.\lambda e\phi.\mathbf{controversial}\,\mathbf{x}e \\
&\qquad \wedge \phi\left(\mathrm{upd}_O(\mathbf{controversial}\,\mathbf{x}e, e)\right)\Big) \\
\widetilde{[\![\textit{'s}]\!]}\ \widetilde{[\![\textit{Ed}]\!]}\ \overline{[\![\textit{claim}]\!]} &\to^*_\beta \lambda \mathbf{P}.\mathbf{P}\Big(\lambda e.\mathrm{sel}\Big(\lambda x.\mathbf{claim}\,x \\
&\qquad \wedge \mathbf{poss}\ \left(\mathrm{sel}(\text{named "Ed"})(\mathrm{upd}_O(\mathbf{claim}\,x, e))\right)x\Big)e\Big) \\
&= \lambda \mathbf{P}.\mathbf{P}\left(\lambda e.\mathrm{sel}\ Q\ e\right)
\end{aligned}$$

where $Q = \Big(\lambda x.\mathbf{claim}\,x \wedge \mathbf{poss}\ \Big(\mathrm{sel}(\text{named "Ed"})\left(\mathrm{upd}_O(\mathbf{claim}\,x, e)\right)\Big)\ x\Big)$. The sentence-

[4] This does not capture the projection properties of supplements; a proposal for acheiving this is presented in Future Work, Section 5.1.1.

[5] The interpretation of the genitive is explained in Lebedeva [2012], Section 5.2.5.

level interpretation of (15) is as follows:

$$\begin{aligned}\mathbf{S}_{(15)} &= \overline{[\![\textit{is highly controversial}]\!]}\left(\widetilde{[\![\textit{'s}]\!]}\ \widetilde{[\![\textit{Ed}]\!]}\ \overline{[\![\textit{claim}]\!]}\right)\\ &\rightarrow^{*}_{\beta} \lambda e\phi.\mathbf{controversial}\,(\mathrm{sel}\ Q\ e) \wedge \phi\left(\mathrm{upd}_{O}\left(\mathbf{controversial}\,(\mathrm{sel}\ Q\ e), e\right)\right)\end{aligned}$$

We want to represent the first context identified by Potts, which we paraphrase as *a claim can be unresearched, a claim is typically controversial* and *an unresearched claim is typically rejected*. We also wish to include the explanatory versions of the last two propositions: *being unresearched is a reason for a claim being controversial* and *being unresearched is a reason for being rejected*. Importantly, in accordance with Pott's description of context, it does not include that *a controversial claim is typically rejected* or that *being controversial is a reason for rejecting a claim*. Let $\mathrm{C} = \langle L, \Delta, \Gamma, \top, \top \rangle$, with L, Δ and Γ as follows:

$$\begin{aligned}L =& \top\\ \Delta =& \forall(\lambda x.\neg\mathbf{researched}\,x \rightarrow \mathbf{controversial}\,x) \wedge\\ & \forall(\lambda x.\neg\mathbf{researched}\,x \rightarrow \mathbf{reject}\,x)\\ \Gamma =& \forall(\lambda x.\neg\mathbf{researched}\,x)\end{aligned}$$

This sentence is to be interpreted in the following discourse:

$$\begin{aligned}\mathbf{D}_0 = \lambda\phi.\exists\Bigg(&\lambda d.\mathrm{named\ ``Ed"}d \wedge \exists\Bigg(\lambda f.\mathbf{claim}\,f \wedge \mathbf{poss}\,d\ f \wedge\\ &\phi\Big(\mathrm{upd}_{\Delta}\left(\mathbf{claim}\,f \wedge \mathbf{poss}\,d\ f, \mathrm{upd}_{\Delta}(\mathrm{named\ ``Ed"}d, \mathrm{C})\right)\Big)\Bigg)\Bigg)\end{aligned}$$

With the update of this background information, the starting context is

$$\begin{aligned}\mathrm{C}_0 &= \mathrm{upd}_{\Delta}\left(\mathbf{claim}\,f \wedge \mathbf{poss}\,d\ f, \mathrm{upd}_{\Delta}(\mathrm{named\ ``Ed"}d, \mathrm{C})\right)\\ &= \langle L, \Delta \wedge \mathrm{named\ ``Ed"}d \wedge \mathbf{claim}\,f \wedge \mathbf{poss}\,d\ f, \Gamma, \top, \top\rangle\end{aligned}$$

Interpreting utterance (15) with respect to context C_0 in discourse D_0:

$$\begin{aligned}
\mathbf{D}_1 &:= \text{dupd } \mathbf{D}_0 \ \mathbf{S}_{(15)} \\
&= \lambda\phi.\mathbf{D}_0(\lambda e.\text{gacc } \mathbf{S}_{(15)} \ e\, \phi) \\
&= \lambda\phi.\left(\lambda\phi.\exists\left(\lambda d.\text{named ``Ed''}d \wedge \exists\left(\lambda f.\mathbf{claim}\, f \wedge \mathbf{poss}\, d\ f \wedge \phi \mathrm{C}_0\right)\right)\right) \\
&\quad\quad (\lambda e.\text{gacc } \mathbf{S}_{(15)} \ e\, \phi) \\
&\to_\beta \lambda\phi.\exists\left(\lambda d.\text{named ``Ed''}d \wedge \exists\left(\lambda f.\mathbf{claim}\, f \wedge \mathbf{poss}\, d\ f \wedge (\lambda e.\text{gacc } \mathbf{S}_{(15)} \ e\, \phi)\mathrm{C}_0\right)\right) \\
&\to_\beta \lambda\phi.\exists\left(\lambda d.\text{named ``Ed''}d \wedge \exists\left(\lambda f.\mathbf{claim}\, f \wedge \mathbf{poss}\, d\ f \wedge \text{gacc } \mathbf{S}_{(15)} \ \mathrm{C}_0\, \phi\right)\right)
\end{aligned}$$

The computation proceeds in the subterm $\mathbf{S}_{(15)}\ \mathrm{C}_0\ \phi$, where the interpretation $\mathbf{S}_{(15)}$ is applied to the context terms and β-reduced to the following term:

$$\begin{aligned}
&\mathbf{S}_{(15)}\ \mathrm{C}_0\ \phi \to^*_\beta \mathbf{controversial}\ \left(\text{sel}\ \left(\lambda x.\mathbf{claim}\, x \wedge \mathbf{poss}\ \left(\text{sel}(\text{named ``Ed''})\mathrm{C}_c\right)\ x\right)\ \mathrm{C}_0\right) \\
&\wedge \phi \left(\text{upd}_O \left(\mathbf{controversial}\ \left(\text{sel}\ \left(\lambda x.\mathbf{claim}\, x \wedge \mathbf{poss}\ \left(\text{sel}(\text{named ``Ed''})\mathrm{C}_c\right)\ x\right)\ \mathrm{C}_0\right), \mathrm{C}_0\right)\right)
\end{aligned}$$

where $\mathrm{C}_c = \left(\text{upd}_O(\mathbf{claim}\, x, \mathrm{C}_0)\right)$. The selection function calls find appropriate referents in the context and since no exceptions are raised, $\mathbf{D}_1$ simplifies to the following term:

$$\begin{aligned}
\mathbf{D}_1 = \lambda\phi.\exists\Big(&\lambda d.\text{named ``Ed''}d \wedge \exists\Big(\lambda f.\mathbf{claim}\, f \wedge \mathbf{poss}\, d\ f \wedge \mathbf{controversial}\, f \wedge \\
&\phi\left(\text{upd}_O\left(\mathbf{controversial}\, f, \mathrm{C}_0\right)\right)\Big)\Big)
\end{aligned}$$

The computation proceeds in the context update to be passed to the continuation of the discourse:

$$\begin{aligned}
\mathrm{C}_1 &:= \text{upd}_O\left(\mathbf{controversial}\, f, \mathrm{C}_0\right) \\
&= \langle L_0, \Delta_0, \Gamma_0, \mathbf{controversial}\, f, \langle E_{\text{res}}\rangle\rangle
\end{aligned}$$

E_{res} is computed according to Definition 3.15 as follows:

$$E_{\text{res}} = D_{\text{res}} \wedge G_{\text{res}}$$
$$D_{\text{res}} = \neg\textbf{researched}\, f \rightarrow \textbf{controversial}\, f$$
$$G_{\text{res}} = \neg\textbf{researched}\, f$$

since D_{res} and G_{res} are instances of Δ_1 and Γ_1 respectively so that $L \wedge D_{\text{res}} \wedge G_{\text{res}}$ is consistent and entails the observation, that is:

$$L \wedge D_{\text{res}} \wedge G_{\text{res}} \vdash \textbf{controversial}\, f$$

To determine the predictions of this context, consider the maximal instances of Δ_1, of which there is only one:[6]

$$\begin{aligned} D_{\text{max}=} & (\neg\textbf{researched}\, d \rightarrow \textbf{controversial}\, d) \wedge (\neg\textbf{researched}\, d \rightarrow \textbf{reject}\, d) \wedge \\ & (\neg\textbf{researched}\, f \rightarrow \textbf{controversial}\, f) \wedge (\neg\textbf{researched}\, f \rightarrow \textbf{reject}\, f) \wedge \\ & \text{named ``Ed''d} \wedge \textbf{claim}\, f \wedge \textbf{poss}\, d\ f \end{aligned}$$

From this we have the following maximal scenario of $(O_1 \wedge G_{\text{res}}, \Delta)$:

$$\begin{aligned} O_1 \wedge G_{\text{res}} \wedge D_{\text{max}} = & \textbf{controversial}\, f \wedge \neg\textbf{researched}\, f \wedge \\ & (\neg\textbf{researched}\, d \rightarrow \textbf{controversial}\, d) \wedge (\neg\textbf{researched}\, d \rightarrow \textbf{reject}\, d) \wedge \\ & (\neg\textbf{researched}\, f \rightarrow \textbf{controversial}\, f) \wedge (\neg\textbf{researched}\, f \rightarrow \textbf{reject}\, f) \wedge \\ & \text{named ``Ed''d} \wedge \textbf{claim}\, f \wedge \textbf{poss}\, d\ f \end{aligned}$$

Since there is only one maximal scenario, and the maximal scenarios generate the extensions, there is only one extension of $(O_1 \wedge G_{\text{res}}, \Delta)$, namely $Th(O_1 \wedge G_{\text{res}}, D_{\text{max}})$. Since this is the only extension and it contains $\textbf{reject}\, d$, it is predicted that Ed's claim should be rejected.

Based on this analysis, and the hypothesis that the conversational implicatures of an utterance are the explanations and predictions of the context in which it is interpreted, (15) can be interpreted as implicating that Ed's claim should be rejected, and that the reason Ed's claim should be rejected is because it is unresearched.

[6] Although this includes the nonsensical instantiations of defaults such as $\neg\textbf{researched}\, d \Rightarrow \textbf{controversial}\, d$, which is interpreted as *if Ed is not researched then usually he is controversial*, since the antecedent would not be uttered in a felicitous conversation, its consequent will not be predicted.

Now consider the utterance including the supplement.

Example 4.10 (*Ed's claim, which is extensively researched, is highly controversial*)**.** The sentence interpretation of (14) uses the interpretations of lexical items from the previous example as well as interpretation (4.2) of the supplemental construction and the following:

$$\begin{aligned}\overline{[\![\textit{is extensively researched}]\!]} &= \lambda \mathbf{X}.\mathbf{X}(\lambda \mathbf{x}.\overline{\mathbf{researched}}\,\mathbf{x}) \\ &\to_\beta \lambda \mathbf{X}.\mathbf{X}\left(\lambda \mathbf{x}.\lambda e\phi.\mathbf{researched}\,\mathbf{x}e \wedge \phi(\mathrm{upd}_O(\mathbf{researched}\,\mathbf{x}e, e))\right)\end{aligned}$$

The interpretation of (14) is given by the following term:

$$\begin{aligned}\mathbf{S}_{(14)} &= \overline{[\![\textit{, which}\ldots\textit{,}]\!]}\left(\widetilde{[\![\textit{'s}]\!]}\,\widetilde{[\![\textit{Ed}]\!]}\,\overline{[\![\textit{claim}]\!]}\right)\left(\overline{[\![\textit{is extensively researched}]\!]}\right)\left(\overline{[\![\textit{is highly controversial}]\!]}\right) \\ &= \lambda \mathbf{xPQ}.(\mathbf{Px} \,\overline{\wedge}\, \mathbf{Qx})\left(\widetilde{[\![\textit{'s}]\!]}\,\widetilde{[\![\textit{Ed}]\!]}\,\overline{[\![\textit{claim}]\!]}\right)\left(\overline{[\![\textit{is extensively researched}]\!]}\right)\left(\overline{[\![\textit{is highly controversial}]\!]}\right) \\ &\to_\beta^* \left(\overline{[\![\textit{is extensively researched}]\!]}\right)\left(\widetilde{[\![\textit{'s}]\!]}\,\widetilde{[\![\textit{Ed}]\!]}\,\overline{[\![\textit{claim}]\!]}\right) \\ &\qquad \overline{\wedge}\,\left(\overline{[\![\textit{is highly controversial}]\!]}\right)\left(\widetilde{[\![\textit{'s}]\!]}\,\widetilde{[\![\textit{Ed}]\!]}\,\overline{[\![\textit{claim}]\!]}\right) \\ &= (\lambda \mathbf{AB}.\mathbf{A}e(\lambda e.\mathbf{B}e\phi))\left(\overline{[\![\textit{is extensively researched}]\!]}\right)\left(\widetilde{[\![\textit{'s}]\!]}\,\widetilde{[\![\textit{Ed}]\!]}\,\overline{[\![\textit{claim}]\!]}\right) \\ &\qquad \left(\overline{[\![\textit{is highly controversial}]\!]}\right)\left(\widetilde{[\![\textit{'s}]\!]}\,\widetilde{[\![\textit{Ed}]\!]}\,\overline{[\![\textit{claim}]\!]}\right) \\ &\to_\beta^* \lambda e\phi.\mathbf{researched}\,\mathrm{sel}\;Q\;e \wedge \mathbf{controversial}\,\mathrm{sel}\;Q\;e_r \wedge \phi\left(\mathrm{upd}_O\left(\mathbf{controversial}\,\mathrm{sel}\;Q\;e_r\right), e_r\right)\end{aligned}$$

where $Q = \left(\lambda x.\mathbf{claim}\,x \wedge \mathbf{poss}\left(\mathrm{sel}(\text{named ``Ed''})\left(\mathrm{upd}_O(\mathbf{claim}\,x, e)\right)\right) x\right)$ and $e_r = \mathrm{upd}_O(\mathbf{researched}\,(\mathrm{sel}\;Q\;e), e)$.

Interpreting in discourse D_0 from Example 4.9, with $\mathrm{C}_0 = \langle L, \Delta \wedge \text{named ``Ed''}d \wedge \mathbf{claim}\,f \wedge \mathbf{poss}\,d\;f, \Gamma, \top, \top\rangle$ as before, we have:

$$\begin{aligned}\mathbf{D}_1 &:= \mathrm{dupd}\;\mathbf{D}_0\;\mathbf{S}_{(14)} \\ &= \lambda\phi.\mathbf{D}_0(\lambda e.\mathrm{gacc}\;\mathbf{S}_{(14)}\;e\,\phi) \\ &\to_\beta^* \lambda\phi.\exists\Big(\lambda d.\text{named ``Ed''}d \wedge \exists\Big(\lambda f.\mathbf{claim}\,f \wedge \mathbf{poss}\,d\;f \wedge \mathrm{gacc}\;\mathbf{S}_{(14)}\;\mathrm{C}_0\,\phi\Big)\Big)\end{aligned}$$

Computation proceeds in the subterm $\mathbf{S}_{(14)}\;\mathrm{C}_0\;\phi$ with evaluation of the sel function calls to determine the pronoun referents:

$$\begin{aligned}\mathbf{S}_{(14)}\ \mathrm{C}_0\ \phi \rightarrow_\beta\ & \lambda e\phi.\mathbf{researched}\ (\mathrm{sel}\ Q\ \mathrm{C}_0) \wedge \mathbf{controversial}\ (\mathrm{sel}\ Q\ \mathrm{C}_r) \wedge \\ & \phi\left(\mathrm{upd}_O\left(\mathbf{controversial}\,\mathrm{sel}\ Q\ \mathrm{C}_r\right), \mathrm{C}_r\right) \\ = & \ \mathbf{researched}\ f \wedge \mathbf{controversial}\ f \wedge \\ & \phi\left(\mathrm{upd}_O\left(\mathbf{controversial}\ f, \mathrm{upd}_O(\mathbf{researched}\ f, \mathrm{C}_0)\right)\right)\end{aligned}$$

where $\mathrm{C}_r = \mathrm{upd}_O(\mathbf{researched}\,(\mathrm{sel}\ \ Q\ \ \mathrm{C}_0), \mathrm{C}_0)$. Consider the following subterm of nested context updates to be passed to the continuation of the discourse:

$$\mathrm{upd}_O\left(\mathbf{controversial}\ f, \mathrm{upd}_O\left(\mathbf{researched}\ f, \mathrm{C}_0\right)\right)$$

Beginning with the inner most context update:

$$\begin{aligned}\mathrm{C}_1 :=\ & \mathrm{upd}(\mathbf{researched}\ f, \mathrm{C}_0) \\ =\ & \langle L_0, \Delta_0, \Gamma_0, \mathbf{researched}\ f, \top\}\rangle\end{aligned}$$

The discourse content is updated but there are no explanations for **researched** f. Regarding predictions, there is the same, single maximal instance of Δ_1 as in the previous example:

$$\begin{aligned}D_{\max =}\ & (\neg\mathbf{researched}\ d \rightarrow \mathbf{controversial}\ d) \wedge (\neg\mathbf{researched}\ d \rightarrow \mathbf{reject}\ d) \wedge \\ & (\neg\mathbf{researched}\ f \rightarrow \mathbf{controversial}\ f) \wedge (\neg\mathbf{researched}\ f \rightarrow \mathbf{reject}\ f) \wedge \\ & \text{named ``Ed''d} \wedge \mathbf{claim}\ f \wedge \mathbf{poss}\ d\ f\end{aligned}$$

The only maximal scenario of (O, Δ) is then $O_1 \wedge D_{\max}$:

$$\begin{aligned}O_1 \wedge D_{\max} =\ & \mathbf{researched}\ f \wedge \\ & (\neg\mathbf{researched}\ d \rightarrow \mathbf{controversial}\ d) \wedge (\neg\mathbf{researched}\ d \rightarrow \mathbf{reject}\ d) \wedge \\ & (\neg\mathbf{researched}\ f \rightarrow \mathbf{controversial}\ f) \wedge (\neg\mathbf{researched}\ f \rightarrow \mathbf{reject}\ f) \wedge \\ & \text{named ``Ed''d} \wedge \mathbf{claim}\ f \wedge \mathbf{poss}\ d\ f\end{aligned}$$

This generates the only extension $Th(O_1, D_{\max})$, which does not include any non-trivial predictions, where a member of $Th(O_1, D_{\max})$ that is a 'trivial' prediction is, for example, a disjunction of propositions, as in $(\neg\mathbf{researched}\ d \rightarrow \mathbf{controversial}\ d) \vee (\neg\mathbf{researched}\ d \rightarrow \mathbf{reject}\ d)$.

For the next context update:

$$\begin{aligned} \mathrm{C}_2 &= \mathrm{upd}(\textbf{controversial}\, f, \mathrm{C}_1) \\ &= \langle L_1, \Delta_1, \Gamma_1, \textbf{researched}\, f \wedge \textbf{controversial}\, f, \top \rangle \end{aligned}$$

The explanation E_{res} from the previous example does not apply as we require an explanation of **researched** $f \wedge$ **controversial** f. In this way, the content introduced in the supplementary construction blocks meaning inferred without the supplement.

4.3.2 Discussion

Potts' meaning – to not dismiss Ed's claim on the basis of being controversial – can be located in the difference between the context with and without the supplementary content. This fits the dynamic conception of meaning as context change potential, with context conceived of as a logic. As in the case of *but*, the extra meaning emerges from reasoning in the context and need not be encoded in a lexical item. This is consistent with Potts' treatment and diagnosis of CI, however suggests that the meaning Potts identifies here is conversational implicature associated with the juxtaposition of content, achieved grammatically by – but not intrinsic to – the supplement construction. It also suggests an account for why alleged conventional implicature content is "extremely hard to articulate" – because it is captured in reasoning rather than a single proposition.

While *but* can be viewed as cancelling a possible conversational implicature, this analysis suggests viewing supplementary content as *preventing* an implicature that could otherwise emerge. As in the case of *but*, it also suggests that the problem of automatically generating discourse context could be addressed by hard-coding the behaviour in the interpretation of the supplement.

It remains to capture the second context in which Potts considers (14), as well as the projection properties of supplements. These problems will be addressed in Sections 5.1.1 and 5.1.4 respectively of the Future Work discussion.

4.4 Summary

Applying the analysis of conversational implicatures by reasoning in the context proves insightful for analysing conventional implicatures, both Gricean and Pottsian. This chapter made the following contributions:

- An update of the treatment of presuppositions in $GL\chi$ according to the new context structure. This suggests a distinction between presuppositions and implicatures based on only the latter involving defeasible reasoning.
- The ability to locate contrasts associated with *but* that exist outside of the two clauses it connects, as in Example 4.1 and the formalization of Bach [1999]'s analysis of *but* in Examples 4.2, 4.3 and 4.5. This demonstrates the formalism's ability to account for interesting puzzles.
- The proposal of a characterization of *but*, in terms of pragmatics, and two infelicity conditions for utterances. The ability to account for infelicitous utterances that have semantically-analogous felicitous utterances is a strength of the framework.
- The delineation of two motivations for a natural language semantics – linguistic enquiry and natural language processing – and the identification of how they can bear on the development of a semantics. The framework as it stands is directed by the first consideration, but an adjustment to favour the latter with automatic context generation was proposed.
- The location of meaning associated with supplementary content in the prevention of conversational implicatures that could be made from the at-issue content on its own. In Example (14), this formalizes the intuition that CIs provide "a clue" as to how the main content should be received.
- An account for Potts' characterization of alleged conventional implicature content as "extremely hard to articulate" by the observation that the meaning Potts is referring to is not given by a single proposition, but rather by a process of reasoning, making it harder to articulate.

Chapter 5

Conclusion

This thesis was motivated by the "still hotly contested" nature of the definitions of presuppositions, conversational implicatures and conventional implicatures, and the suggestion of moving from "splitting and lumping" to developing "rich theories of properties like 'conventional', 'backgrounded', and 'projective', the way those properties interact and the effects of those interactions on language and cognition" (Potts [2015]). The intention was to contribute to "rich theories" by capturing meaning associated with these categories in a single formal framework, allowing for formal comparisons of their properties while accounting for the debate surrounding their definitions.

To form a theory requires accounting for substantially more instances of implicatures than in this work, which uses very limited data. This thesis does, however, make a case for extending the popular characterization of *conversational implicatures as abductions* to *conversational implicatures as defeasible inferences* more generally, and demonstrates that this can be applied to account for meaning associated with both Gricean and Pottsian conventional implicatures. This forms a common basis for investigating relationships to rich theories of properties. It also provides an account for the existence of debate around these meaning classes in that much of the meaning is better captured in a process of reasoning, rather than in the traditional form of individual propositions.

Another finding is the effect of motivation on the development of a formalism. It was observed that prioritizing automatic generation leads to a classical theory of *but* in *GL*χ, while taking the minimal approach of avoiding encoded meaning into lexical items still allows *but* to be captured, albeit in a way that corresponds to a very different theory of meaning.

5.1 Future Work

This thesis concludes with a discussion of further directions for analysis in – and development of – the framework.

5.1.1 Projection Problem for Supplements

Supplementary content has very similar projection properties to presupposition. However, given that the epistemic status of supplementary content is different to that of presuppositions, in the sense that the content is not backgrounded, their projection cannot necessarily be accounted for in the same way as that of presuppositions: "The challenge for a formal theory of projection phenomena, now, is to account for the projection behaviour of presuppositions and CIs in a unified way, while appreciating their information structural difference" [Venhuizen et al., 2014, p.64].

One proposal for addressing projection for supplements is type-raising – using continuations to change the order of evaluation. This is not just ad hoc – it is principled, justified by analogy with type raising of quantified noun phrases in Section 2.2.3. Quantified noun phrases like *everybody*, *nobody* and *somebody* have type $(\iota \to o) \to o$, where $\iota \to o$ is thought of as the set of entities satisfying a particular property, and $(\iota \to o) \to o$ is a set of properties of individuals. In the case of supplements, they are described as altering how a sentence is interpreted – a sentence modifier or property of a sentence. By type raising a sentence to type $o \to o$, we consider the set of propositions satisfying a particular property – that given in the supplement.

To demonstrate how this could work, consider the following utterance.

(16) It is not the case that John, who loves Mary, smiles at her.

The desired interpretation evaluates "smiles at her" in a context containing "John loves Mary" to find the correct referent for "her", with only "John smiles at her" being negated.

Example 5.1 (*It is not the case that John, who loves Mary, smiles at her*, sentence interpretation)**.** For clarity, let $\mathrm{sel}_{\mathrm{M}}$ abbreviate $\mathrm{sel}(\lambda x.(\text{named "Mary"})e)$, $\mathrm{sel}_{\mathrm{J}}$ abbreviate $\mathrm{sel}(\lambda x.(\text{named "John"})e)$ and $\phi\ (p \bullet e)$ abbreviate $\phi\left(\mathrm{upd}\ (p, e)\right)$. Then we want the reduced form of the interpretation of (16) to be the following term:

$$\lambda e\phi.\mathbf{lv}\,(\mathrm{sel_J}e)(\mathrm{sel_M}e) \wedge \neg\mathbf{sm}\,\left(\mathrm{sel_J}\,(\mathbf{lv}\,(\mathrm{sel_J}e)(\mathrm{sel_M}e)\bullet e)\right)\left(\mathrm{sel_h}\,(\mathbf{lv}\,(\mathrm{sel_J}e)(\mathrm{sel_M}e)\bullet e)\right)$$
$$\wedge\,\phi\,(\mathbf{lv}\ (\mathrm{sel_J}e)(\mathrm{sel_M}e)\bullet e)$$

Consider the following variation $\star$ of dynamic conjunction, originally defined in 2.9f, and $\bar{\neg}_\star$ of dynamic negation originally defined in 2.9g:

$$\star = \lambda\mathbf{AB}.\lambda\kappa.\lambda e\phi\mathbf{A}e(\lambda e.\kappa\mathbf{B}e\phi)$$
$$\bar{\neg}_\star = \lambda\kappa.\kappa\left(\lambda\mathbf{A}.\lambda e\phi.\neg(\mathbf{A}e(\lambda e.\top))\wedge\phi e\right)$$

The following term is the interpretation of (16), with $\star$ and $\bar{\neg}_\star$ together allowing the supplement to escape negation:

$$\bar{\neg}_\star\left(\star\left(\overline{\mathbf{lv}}\,\mathrm{sel_J sel_M}\right)\left(\overline{\mathbf{sm}}\,\mathrm{sel_J sel_h}\right)\right)$$
$$=\left(\lambda\kappa.\kappa\left(\lambda\mathbf{A}.\lambda e\phi.\neg(\mathbf{A}e(\lambda e.\top))\wedge\phi e\right)\right)\left(\star\left(\overline{\mathbf{lv}}\,\mathrm{sel_J sel_M}\right)\left(\overline{\mathbf{sm}}\,\mathrm{sel_J sel_h}\right)\right)$$
$$\to_\beta\left(\star\left(\overline{\mathbf{lv}}\,\mathrm{sel_J sel_M}\right)\left(\overline{\mathbf{sm}}\,\mathrm{sel_J sel_h}\right)\right)\left(\lambda\mathbf{A}.\lambda e\phi.\neg\left(\mathbf{A}e(\lambda e.\top)\right)\wedge\phi e\right)$$
$$=\left(\lambda\mathbf{AB}.\lambda\kappa.\lambda e\phi\mathbf{A}e(\lambda e.\kappa\mathbf{B}e\phi)\left(\overline{\mathbf{lv}}\,\mathrm{sel_J sel_M}\right)\left(\overline{\mathbf{sm}}\,\mathrm{sel_J sel_h}\right)\right)\left(\lambda\mathbf{A}.\lambda e\phi.\neg\left(\mathbf{A}e(\lambda e.\top)\right)\wedge\phi e\right)$$
$$\to_\beta^*\lambda\kappa.\lambda e\phi\left(\overline{\mathbf{lv}}\,\mathrm{sel_J sel_M}\right)e\left(\lambda e.\kappa\left(\overline{\mathbf{sm}}\,\mathrm{sel_J sel_h}\right)e\phi\right)\left(\lambda\mathbf{A}.\lambda e\phi.\neg\left(\mathbf{A}e(\lambda e.\top)\right)\wedge\phi e\right)$$
$$\to_\beta\lambda e\phi\left(\overline{\mathbf{lv}}\,\mathrm{sel_J sel_M}\right)e\left(\lambda e.\left(\lambda\mathbf{A}.\lambda e\phi.\neg\left(\mathbf{A}e(\lambda e.\top)\right)\wedge\phi e\right)\left(\overline{\mathbf{sm}}\,\mathrm{sel_J sel_h}\right)e\phi\right)$$
$$\to_\beta\lambda e\phi\left(\overline{\mathbf{lv}}\,\mathrm{sel_J sel_M}\right)e\left(\lambda e.\left(\lambda e\phi.\neg\left(\left(\overline{\mathbf{sm}}\,\mathrm{sel_J sel_h}\right)e(\lambda e.\top)\right)\wedge\phi e\right)e\phi\right)$$

Performing reductions on the subterm:

$$\lambda e.\left(\lambda e\phi.\neg\left(\left(\overline{\mathbf{sm}}\,\mathrm{sel_J sel_h}\right)e(\lambda e.\top)\right)\wedge\phi e\right)e\phi$$
$$=\lambda e.\left(\lambda e\phi.\neg\left(\left(\lambda e\phi.\mathbf{sm}\,(\mathrm{sel_J}e)(\mathrm{sel_h}e)\wedge\phi\left(\mathbf{sm}\,(\mathrm{sel_J}e)(\mathrm{sel_h}e)\bullet e\right)\right)e(\lambda e.\top)\right)\wedge\phi e\right)e\phi$$
$$\to_\beta^*\lambda e.\left(\lambda e\phi.\neg\left(\mathbf{sm}\,(\mathrm{sel_J}e)(\mathrm{sel_h}e)\wedge(\lambda e.\top)\left(\mathbf{sm}\,(\mathrm{sel_J}e)(\mathrm{sel_h}e)\bullet e\right)\right)\wedge\phi e\right)e\phi$$
$$=\lambda e.\left(\lambda e\phi.\neg\mathbf{sm}\,(\mathrm{sel_J}e)(\mathrm{sel_h}e)\wedge\phi e\right)e\phi$$
$$\to_\beta^*\lambda e.\neg\mathbf{sm}\,(\mathrm{sel_J}e)(\mathrm{sel_h}e)\wedge\phi e$$

Substituting the reduced subterm back into the complete term:

$$\lambda e\phi.\left(\overline{\mathbf{lv}}\,\mathrm{sel_J sel_M}\right) e \left(\lambda e.\left(\lambda e\phi.\neg\left(\left(\overline{\mathbf{sm}}\,\mathrm{sel_J sel_h}\right) e(\lambda e.\top)\right) \wedge \phi e\right) e\phi\right)$$

$$= \lambda e\phi.\left(\overline{\mathbf{lv}}\,\mathrm{sel_J sel_M}\right) e \left(\lambda e.\neg\mathbf{sm}\,(\mathrm{sel_J} e)(\mathrm{sel_h} e) \wedge \phi e\right)$$

$$= \lambda e\phi.\left(\lambda e\phi.\mathbf{lv}\,(\mathrm{sel_J} e)(\mathrm{sel_M} e) \wedge \phi\left(\mathbf{lv}\,(\mathrm{sel_J} e)(\mathrm{sel_M} e) \bullet e\right)\right) e \left(\lambda e.\neg\mathbf{sm}\,(\mathrm{sel_J} e)(\mathrm{sel_h} e) \wedge \phi e\right)$$

$$\to^*_\beta \lambda e\phi.\mathbf{lv}\,(\mathrm{sel_J} e)(\mathrm{sel_M} e) \wedge \left(\lambda e.\neg\mathbf{sm}\,(\mathrm{sel_J} e)(\mathrm{sel_h} e) \wedge \phi e\right)\left(\mathbf{lv}\,(\mathrm{sel_J} e)(\mathrm{sel_M} e) \bullet e\right)$$

$$\to_\beta \lambda e\phi.\mathbf{lv}\,(\mathrm{sel_J} e)(\mathrm{sel_M} e) \wedge \neg\mathbf{sm}\left(\mathrm{sel_J}\left(\mathbf{lv}\,(\mathrm{sel_J} e)(\mathrm{sel_M} e) \bullet e\right)\right)\left(\mathrm{sel_h}\left(\mathbf{lv}\,(\mathrm{sel_J} e)(\mathrm{sel_M} e) \bullet e\right)\right)$$
$$\wedge\, \phi\left(\mathbf{lv}\;(\mathrm{sel_J} e)(\mathrm{sel_M} e) \bullet e\right)$$

This suggests investigating the use of continuations to capture the projection properties of supplements.

5.1.2 Properties of Meaning Classes

With the formalization of certain presuppositions, conversational implicatures and conventional implicatures in the same framework, they can be formally compared in terms of their properties. We proceed by discussing those properties that can now be characterized in $GL\chi$ with the new context structure, as well as those that remain to be considered.

Backgrounded is a property referring to meaning that is assumed by the speaker to be part of the common context. It is associated with presupposed content and with the new context structure, corresponds naturally to information in the set Δ of defaults and background knowledge.

Meaning is said to be *conventional* if it is lexically encoded, that is, triggered by a lexical item. Meaning that is not conventional is not tied to a particular lexical item, rather it is dependent on the context that it is interpreted in. This has a straightforward characterization in $GL\chi$, since there is clear separation between content and context.

Meaning associated with an utterance is said to be *cancellable*, or deniable, if it is felicitous to follow with an utterance negating this meaning. For example:

(17) a) Ed has six fingers – in fact he has ten.

b) # Ed has exactly six fingers – in fact he has ten. [Potts, 2006, p.7]

Cancellability is easy to characterize once defeasible and non-defeasible inferences are delineated – defeasible inference may be cancelled, non-defeasible inference may not.

5.1.3 Speaker-Relative Context and Disagreements

Capturing discourse containing disagreements will introduce inconsistencies into the context. An example of this is Bach [1999]'s demonstration of the context-dependence of *but* by interpreting (7) in the following discourse:

(18) A: Shaq is huge and clumsy.
B: Shaq is huge but he is agile.

Here the contrast exists between Shaq being clumsy and Shaq being agile, rather than between Shaq being huge and Shaq being agile.

One way of delaying with this is to impose an assumption of internal consistency for each speaker, while permitting contradiction between speakers, by parametrizing context contributions with respect to the speaker.

5.1.4 Cognitive Modelling

The scope of the framework could be increased by incorporating a simple model of cognition, as in [Asher and Lascarides, 2003, Chapter 9]. For example, the conversational implicatures in discourse (9)a could be more accurately modelled as "speaker A *believes* that Robinson should be invited, and *believes* that he is in Oxford, which means he is not in America". It would also allow the second context of (14), given in Section 4.3, to be captured by formalizing the goal of "a group of detractors of Ed's claim".

Beyond the contributions of this thesis, several avenues present themselves for using the context logic in $GL\chi$ to further our understanding of the meaning classes of implicatures and presuppositions.

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